Available online at www.sciencedirect.com
ScienceDirect
Physica D 231 (2007) 108–115
PHYSICA D
www.elsevier.com/locate/physd

Algorithms for generating surrogate data for sparsely quantized time series

Tomoya Suzuki[a,*], Tohru Ikeguchi[b], Masuo Suzuki[c]

[a] *Department of Information Systems Design, Doshisha University, 1-3 Tatara-Miyakodani, Kyotanabe-city, Kyoto, 610-0394, Japan*
[b] *Graduate School of Science and Engineering, Saitama University, 225 Shimo-Ohkubo, Sakura-ku, Saitama-city, Saitama, 338-8570, Japan*
[c] *Graduate School of Science, Tokyo University of Science, 1-3 Kagurazaka, Shinjuku-ku, Tokyo, 162-8601, Japan*

Received 15 December 2006; accepted 23 April 2007
Available online 7 May 2007

Communicated by C.K.R.T. Jones

Abstract

The method of surrogate data is frequently used for a statistical examination of nonlinear properties underlying original data. If surrogate data sets are generated by a null hypothesis that the data are derived by a linear process, a rejection of the hypothesis means that the original data have more complex properties. However, we found that if an algorithm for generating surrogate data, for example, amplitude adjusted Fourier transformed, is applied to sparsely quantized data, there are large discrepancies between their power spectrum and that of the original data in lower frequency regions. We performed some simulations to confirm that these errors often lead to false rejections.

In this paper, in order to prevent such drawbacks, we advance an extended hypothesis, and propose two improved algorithms for generating surrogate data that reduce the discrepancies of the power spectra. We also confirm the validity of the two improved algorithms with numerical simulations by showing that the extended null hypothesis can be rejected if the time series is produced from chaotic dynamical systems. Finally, we applied these algorithms for analyzing financial tick data as a real example; then we showed that the extended null hypothesis cannot be rejected because the nonlinear statistics or nonlinear prediction errors exhibited are the same as those of the original financial tick time series.

Keywords: The method of surrogate data; False rejection; Quantized data; Econophysics

1. Introduction

Real time series data often show complex behavior. This complexity might be produced by either a stochastic mechanism or a nonlinear deterministic, possibly chaotic mechanism. If it is produced from a linear stochastic mechanism, its power spectrum is the only information for understanding the system, and more detailed information does not exist. However, if the system is derived from a nonlinear deterministic dynamical system, it is necessary to understand the system in greater detail for predicting and controlling such complex phenomena [1] from information other than power spectra. Actually, nonlinearity is often an essential origin for complex behavior. Thus, it is important to examine whether nonlinearity exists through time series analysis.

For nonlinear time series analysis [2], the method of surrogate data [3] is often used to test analyzed results that may be spurious due to artifacts such as a smaller number of data points and lower qualities of measurements in the original data. Surrogate time series is a shuffled time series of the original data, but it preserves some statistical properties of the original data such as empirical histograms and/or power spectra. By comparing the properties of the original data with those of surrogate data sets from the viewpoint of nonlinear statistics, we can discuss whether the original data have the nonlinear property.

In actual analysis, all experimental data are discretized (sampled) and quantized (digitized) to a greater or lesser extent. In particular, we often have to analyze coarsely quantized time series data because the resolution for representing time series values is poor. In addition, any modern time series analysis methods using digital computers must always quantize the observed analog time series data, for example, using an analog-to-digital converter, before inputting them into digital computers. In other words, even if state values are generated from a smooth dynamical system, which makes them

* Corresponding author.
E-mail address: tsuzuki@mail.doshisha.ac.jp (T. Suzuki).

0167-2789/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.physd.2007.04.006

continuous, numerical analyses always require quantized values of time series data.

A typical example of the coarsely quantized time series is a financial time series that fluctuates within finite decimal points. In fact, financial data are often analyzed using the nonlinear dynamical system theory to investigate the possibility that it has nonlinearity [4]. In particular, the method of surrogate data is often used to evaluate possible nonlinearity in financial data [5]. Thus, it is very important to clarify whether and how the coarse quantization in an observed time series affects analysis results by using the method of surrogate data.

In the present paper, we show that a false rejection occurs if we blindly apply the amplitude adjusted Fourier transformed (AAFT) algorithm [3,6] to such coarsely quantized time series data. If a false rejection occurs in analyzing a stochastic linear time series, the surrogate test concludes that the stochastic linear time series has more complex properties than a corresponding null hypothesis. Thus, such false rejections would lead to a fatal conclusion.

In the following sections, in order to resolve the issue, we first advance a null hypothesis for such a coarsely quantized time series, and propose two corresponding algorithms, which are modifications of the original AAFT surrogate. Secondly, we evaluate the validity of the two modified algorithms for coarsely quantized time series data via numerical simulations. Finally, we apply these algorithms to financial data as an example of real data analysis in order to investigate whether the coarse quantization might affect the analysis [4,5].

2. The method of surrogate data

The method of surrogate data is useful for obtaining reliable results in nonlinear time series analysis [3]. Namely, it avoids spurious identification of deterministic chaos underlying time series data through a careless estimation of nonlinear indices such as fractal dimensions [7] or Lyapunov exponents [8]. Surrogate time series is a stochastic time series, and it is constructed by satisfying a null hypothesis based on some linear and stochastic characteristics. By comparing the nonlinear indices of the observed data and surrogate data sets, it is possible to decide whether a linear process is sufficient for describing the observed data. If the observed data do not have nonlinear deterministic properties but have linear ones, the estimated properties become almost the same as those of its surrogates. On the other hand, if the original data cannot be described using a linear system, the properties are completely different from those of the surrogates.

In the present paper, we primarily aim to develop an algorithm for generating surrogate data [3], where the null hypothesis is that the original data are generated by a linear stochastic process; however, the original data are observed through a static monotonic nonlinear transformation with a coarse quantization. In order to develop the algorithm, we extended the algorithm for generating amplitude adjusted Fourier transform (AAFT) surrogate data [3,6]. The AAFT surrogate data are generated by the following steps:

1. We produce Gaussian random time series $\eta(t) \sim N(0, 1)$ and shuffle the temporal order to obtain the same rank order as with the original time series $X(t)$. Here, the rank order is defined as an order of state values of a time series [3]. In this process, the shuffled time series $\omega(t)$ corresponds to a linear stochastic process and $X(t)$ is considered to be a realization of a transformation of $\omega(t)$ through a static monotonic nonlinear function $h(\cdot)$. That is, $X(t) = h(\omega(t))$.
2. Since the shuffled time series $\omega(t)$ is a realization that can be characterized only by its power spectrum, we apply the Fourier transformed (FT) algorithm [3] to $\omega(t)$. The FT algorithm is as follows: the Fourier transform is applied to a target time series, in this case $\omega(t)$, and its power spectrum is obtained. Next, preserving a symmetry of phase components of the Fourier transform, the phase components are randomized. Then, the inverse Fourier transform generates an FT surrogate, which preserves the power spectrum of the target time series $\omega(t)$. Namely, in this process, we make an FT surrogate $\hat{\omega}(t)$ of $\omega(t)$, which has exactly the same power spectrum as $\omega(t)$.
3. We shuffle the original data $X(t)$ to obtain the same rank order as the FT surrogate data $\hat{\omega}(t)$ given in the previous step. Here, we again consider a static monotonic nonlinear function $h(\cdot)$. Namely, $\hat{X}(t) = h(\hat{\omega}(t))$, and $\hat{X}(t)$ is called the AAFT surrogate data version of $X(t)$.

The AAFT surrogate $\hat{X}(t)$ completely preserves the empirical histogram and approximately preserves the power spectrum of the original data $X(t)$.

For preserving the power spectrum of the original time series, we have other algorithms besides the AAFT—the Fourier shuffle (FS) algorithm [3,9] and the iterative amplitude adjusted Fourier transformed (IAAFT) [3,10] algorithm. Although these algorithms for generating surrogate data often exhibit better abilities for preserving the original power spectrum than the AAFT, we considered an extension of the null hypothesis of the AAFT algorithm in the present paper. The reason is that it is still important to test the extended null hypothesis explicitly using an algorithm which is based on the extended null hypothesis. It is true that if we use an algorithm for producing the IAAFT [3,10] or the FS surrogates [3,9], we can generate surrogate data having the same power spectrum as the original data. Even so, a hypothesis testing using the extended algorithm of the AAFT surrogate is still important, because we can explicitly test the null hypothesis in a more direct way.

3. Modification of the surrogate method

In this section, we discuss the application of an AAFT surrogate algorithm to a coarsely quantized time series. As introduced in Section 2, the hypothesis of the AAFT surrogate has a static monotonic nonlinear function $h(\cdot)$, which is used for $X(t) = h(\omega(t))$ and $\hat{X}(t) = h(\hat{\omega}(t))$. In the present paper, we generalized the hypothesis of the function $h(\cdot)$ as a static monotonic nonlinear function including its digitized form. By using the generalized hypothesis, we can perform the AAFT surrogate test for a coarsely quantized time series

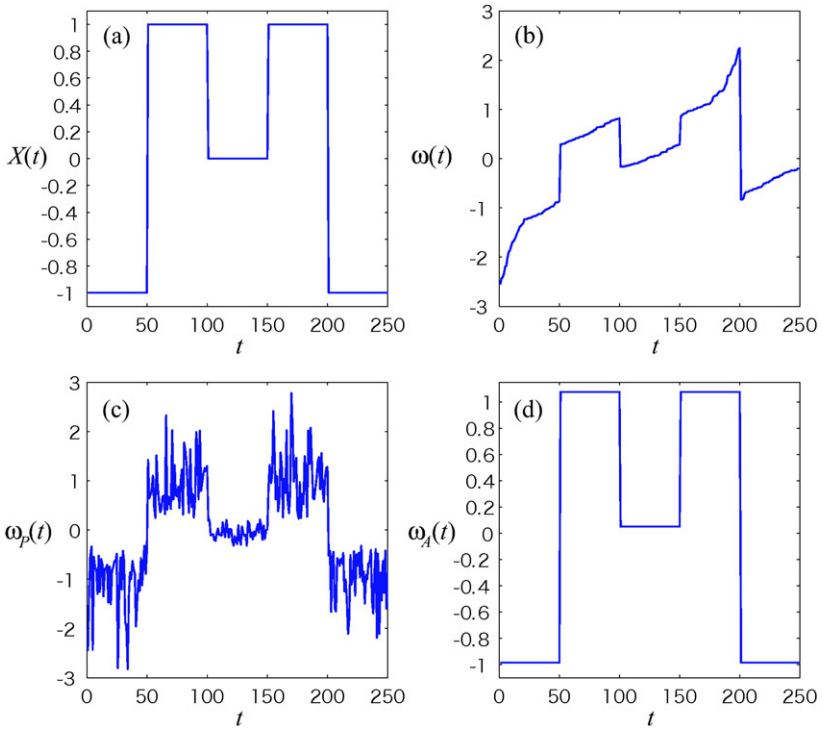


Fig. 1. (a) An example of an original time series $X(t)$. The time series $\omega(t)$ in (b) is generated by the first process of creating the original AAFT [3]. The time series $\omega_P(t)$ in (c) and $\omega_A(t)$ in (d) are generated by the two improved algorithms: the P-AAFT and A-AAFT algorithms, respectively. We found that our proposed algorithms modified the blurred spurious shape of the time series $\omega(t)$ produced by the original algorithm for creating AAFT.

with the steps 1–3 of the AAFT algorithms, just introduced in Section 2. The important point is not to apply the AAFT algorithm carelessly, especially when deciding the rank order, because it might generate an incorrect surrogate time series. Subsequently, we discuss a more careful way of ordering the rank order between $X(t)$ and $\omega(t)$ ($\hat{X}(t)$ and $\hat{\omega}(t)$).

3.1. Problems in deciding a rank order

In the case of applying the original AAFT algorithm to a coarsely quantized time series, a severe problem exists in deciding its rank order; an ordinary sorting algorithm defines earlier temporal indices with the same state value to have a lower rank order. Here, the essential point is that the wave form of $\omega(t)$, produced by the first step of generating AAFT surrogate data, has a different form to the original data $X(t)$, as shown in Fig. 1(a) and (b). In particular, the time series $\omega(t)$ in Fig. 1(b) shows a large discrepancy from the original in low frequency regions, and thus its power spectrum might differ from the original one. Then, it is natural to expect the AAFT surrogate data not to preserve the power spectrum and linear property of $X(t)$. Thus, it is possible that the null hypothesis is rejected even if the hypothesis is correct for the original data.

3.2. Modified algorithms

In order to eliminate the source of a false rejection, we modified the method for deciding the rank order between $X(t)$ and $\omega(t)$. In the present paper, we discuss two modifications.

First, to prevent loss of information of the same state values, we added uniform random numbers of a very small level to the original data $X(t)$ in order to shuffle the rank orders of the state values. Here, the dynamic range of the uniform random number is between zero and $\min_t\{\Delta X(t)\}$, where $\Delta X(t) = X(t) - X(t-1)$. We call this the P (perturbed)-AAFT algorithm. Fig. 1(c) shows the time series $\omega_P(t)$ generated by the above procedure against $\omega(t)$ in the first process of generating AAFT surrogate data. Although this time series includes higher frequency components, the difference can be canceled by shuffling the discretized original data $X(t)$ in the third process of generating AAFT surrogate data. However, the shuffling could not cancel the low frequency difference of the AAFT surrogates, as shown in Fig. 1(b).

Second, we recorded each group having the same state values of the original data $X(t)$. Then, each set of $\omega(t)$ having the same rank order as each group of $X(t)$ is averaged. We call this the A (averaged)-AAFT algorithm. Fig. 1(d) shows the time series $\omega_A(t)$ improved by the A-AAFT algorithm. Although the A-AAFT algorithm is considerably more expensive than the P-AAFT algorithm, because it searches for groups having the same state value, the time series wave form $\omega_A(t)$ is almost the same as the original $X(t)$; using the A-AAFT algorithm might preserve the original power spectrum more accurately than using the P-AAFT algorithm.

To produce a surrogate time series, the P-AAFT and A-AAFT surrogates also follow the same second and third steps of the AAFT algorithm introduced in Section 2 with the improved $\omega(t)$, that is, $\omega_P(t)$ and $\omega_A(t)$.

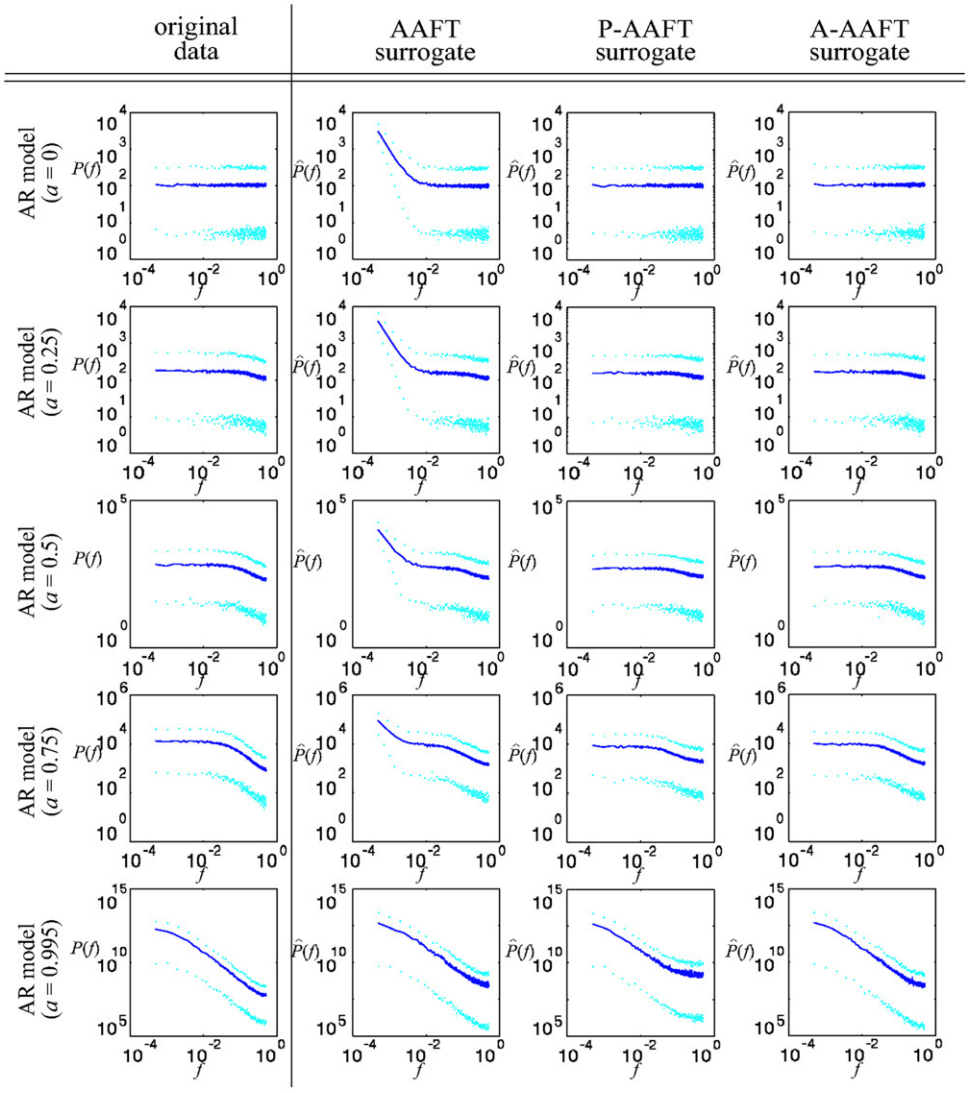


Fig. 2. Power spectra of the original data, ordinary AAFT, P-AAFT, and A-AAFT surrogate data in the case that $q = 7$ bits, $N = 2048$ and $d = 3$. In each plot, solid lines are averages of 500 trials, and dotted lines show 5% confidence intervals. The horizontal axes are frequency f, and the vertical axes are power spectrum $P(f)$.

4. Simulations

False rejection is a result of the difference between the power spectrum of the original data and that of the surrogate data. Then, in order to evaluate the performance of surrogate algorithms, we investigated the discrepancy of the power spectra with respect to the original data one by calculating the normalized mean square error:

$$E = \frac{\sum_f (P(f) - \hat{P}(f))^2}{\sum_f (P(f) - \bar{P}(f))^2}, \tag{1}$$

where $P(f)$ is the power spectrum of the original data; $\hat{P}(f)$ the power spectrum of its surrogate; $\bar{P}$ the average of $P(f)$; and f a frequency component. We then used numerical data and real data for evaluating E and for performing surrogate tests.

4.1. Numerical data: AR model and Ikeda map

For simulations, we used an autoregressive (AR) model [1]:

$$x(t+1) = ax(t) + \eta(t), \tag{2}$$

where $\eta(t)$ is a Gaussian random number with zero mean and unit variance. In the present paper, we set $a < 1$ to satisfy the stationarity condition, that is, the null hypothesis of the original AAFT surrogate method. Next, to follow the null hypothesis, we transform $x(t)$ into $W(t) = x(t) \times |x(t)|^d$, which is a monotonic nonlinear functions [10]. Outputs from the monotonic nonlinear function, $W(t)$, correspond to the observed data. However, since we would like to discuss whether the original data are coarsely quantized, we quantized $W(t)$ using q bits to obtain the observed data $X(t)$. In Fig. 2, we show the power spectra of the original AAFT, P-AAFT, and A-AAFT surrogate data of $X(t)$, whose data length is N, when $q = 7$ bits, $N = 2,048$, and $d = 3$.

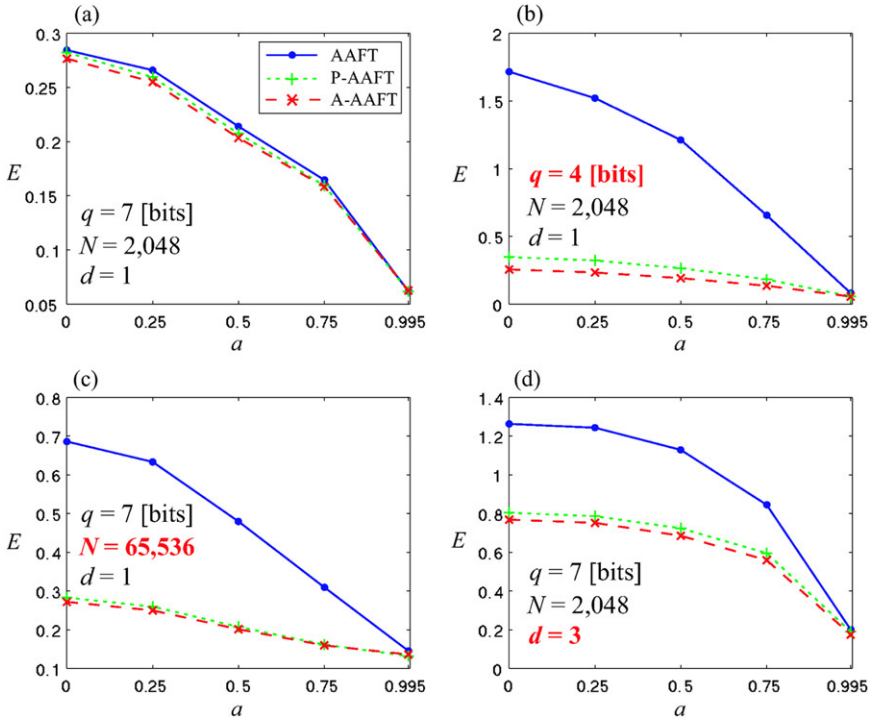


Fig. 3. The characteristic property of E for several parameters. Each E is estimated by averaging 500 trials. The variable a is the coefficient of the AR model in Eq. (2). Although (a) shows that the differences among three types of AAFT algorithms were not significant, changing one of the parameters increased the differences, as shown in (b), (c).

We found that the AAFT surrogates have considerably larger powers in the lower frequency regions than the original data. However, the power spectra of P-AAFT and A-AAFT exhibit good results because they preserved the statistical features of the original data more accurately.

In order to investigate the characteristics of E with several parameters, we varied the parameters q, N, and d. In each situation, we simulated 500 trials of an AR model and its surrogate data sets, and then calculated each E using Eq. (1). Fig. 3 shows the average values of E. By comparing Fig. 3(a) with (b), we confirmed that the advantage of P-AAFT and A-AAFT disappeared gradually as the quantization level q increased. By comparing Fig. 3(a) with (c) or (d), if the data length N is large as shown in Fig. 3(c) or the nonlinear level d is higher as shown in Fig. 3(d), we confirmed the advantage again. As discussed in Section 3.2, in the third process for generating $\hat{X}(t)$ by shuffling $X(t)$ following the rank order of $\hat{\omega}(t)$, the high frequency components of P-AAFT (Fig. 1(c)) did not make the performance poorer (dashed lines with plus signs in Fig. 3(c) and (d)); however, the low frequency components that were not removed by AAFT (Fig. 1(b)) affected the performance (solid lines in Fig. 3(c) and (d)). However, in any situation, the A-AAFT algorithm stably shows a better performance than the P-AAFT algorithm.

Next, we show the results of surrogate tests with an AR model in Fig. 4(a) and (b). For each test, 19 surrogates were created using the AAFT, P-AAFT, and A-AAFT algorithms introduced in Section 3. When we used considerably quantized data, we were unable to reliably estimate nonlinear indices such as correlation dimensions [2]. Instead, we used a local linear approximation prediction method for the data embedded in a D-dimensional delay space [2]. We used the first half of a time series for learning, and then we predicted its second half for evaluation. Here, we predicted D future steps because the same values tend to continue in quantized time series. Then, we calculated the normalized mean square error as the prediction error:

$$E = \frac{\sum_t (X(t) - \hat{X}(t))^2}{\sum_t (X(t) - \bar{X}(t))^2}, \tag{3}$$

where $X(t)$ is the original time series, $\bar{X}$ is the average of $X(t)$, and $\hat{X}(t)$ is the predicted time series. The null hypothesis is rejected at 95% level of significance on the basis of two-tailed tests. If the prediction error for the original data is smaller than that for the 19 surrogates, this is defined as *under*-rejection; if the prediction error for the data is larger than that for the 19 surrogates, this is defined as *over*-rejection. To estimate each rejection rate, we performed 200 independent tests. Because the original data are from the AR model, any rejection is a false rejection.

Fig. 4(a) shows the results of surrogate tests with the AR model in the case of $q = 4$ bits, $d = 3$, $N = 2048$, and $D = 5$. Except for $a = 1$, the under-rejection rates of AAFT surrogates are larger because the AAFT surrogate data have false low frequency components. Namely, this false low frequency, that is, false long-term memory, makes the prediction error small unfairly. On the other hand, the over-rejection rates of AAFT surrogates are as small as those of the proposed surrogates. If $a = 1$, the AR model is not stationary and the rejection is a

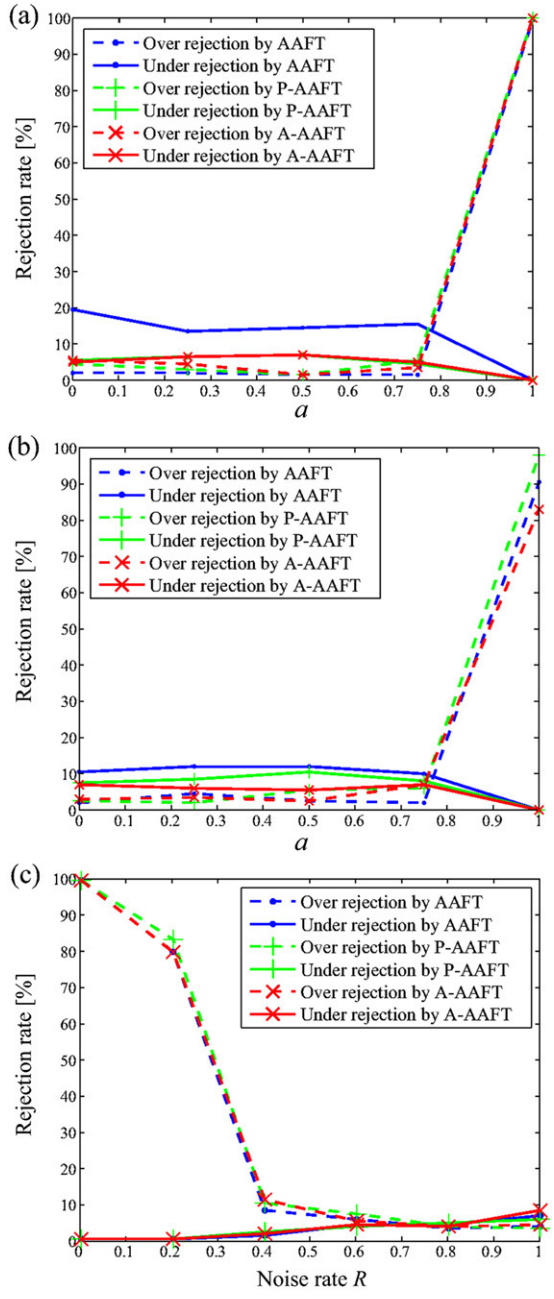


Fig. 4. The rates of rejection, over-rejection, and under-rejection, by surrogate tests for (a) the AR model ($q = 4$, $d = 3$, and $N = 2048$), and (b) the AR model ($q = 7$, $d = 3$, and $N = 2048$). (c) The Ikeda map ($q = 4$ and $N = 2048$) disturbed by a Gaussian random noise. Each data set was embedded in $D = 5$-dimensional state space for prediction. In (a) and (b), a is the coefficient of the AR model.

true one. Thus, each performance of three surrogate methods is valid even if $a = 1$. In any case, the proposed surrogates show a good performance in the reliable surrogate test. Fig. 4(b) is the same as Fig. 4(a) but with $q = 7$ bits. The over-rejection rates of AAFT surrogates are reduced because the observed data $X(t)$ become smoother. However, the proposed surrogates are superior to the ordinary AAFT surrogate as well as the result shown in Fig. 4(a).

To examine whether the proposed surrogates reduce the power of the surrogate test, we used the Ikeda map [2]:

$$\begin{cases} x_1(t+1) = q + b(x_1(t)\cos(\theta(t)) - x_2(t)\sin(\theta(t))) \\ x_2(t+1) = b(x_1(t)\sin(\theta(t)) + x_2(t)\cos(\theta(t))) \end{cases} \quad (4)$$

where $\theta(t) = \kappa - \alpha/(1 + x^2(t) + y^2(t))$. We set $q = 1.0$, $b = 0.9$, $\kappa = 0.4$, and $\alpha = 6.0$.

For simulations, to consider the case where the nonlinearity is disturbed by the observation noise, we disturbed the first variable $x_1(t)$ of the Ikeda map ($N = 2048$) using Gaussian random noise with zero mean. Here, we defined the noise rate R as the ratio of the variance of the Gaussian random noise to that of the original time series $x_1(t)$. Then, we quantized the time series by $q = 4$ to obtain a coarsely quantized time series $X_1(t)$.

Fig. 4(c) shows the rejection rates for the surrogate tests for the Ikeda map $x_1(t)$ obtained by using AAFT, P-AAFT, and A-AAFT algorithms. For surrogate tests, we estimated the prediction error of each data set embedded in $D = 5$-dimensional state space for prediction. As results, we can confirm that using the proposed algorithms, P-AAFT and A-AAFT, does not reduce the power of the AAFT surrogate test because the rejection rates for each of the algorithms are almost the same for all noise rates.

4.2. Application to real data analysis

Investigating real data and deriving heuristic laws from them is very effective for understanding the mechanism of a system. Recently, in the academic field of econophysics [4], such merits of real data analysis have been emphasized. Some theories on a financial system have been derived by such strategies, and mechanisms have been described using heuristic laws rather than theoretical suppositions. Thus, reliable analytic tools are desired for such studies. However, every financial time series moves within finite decimal points; careless use of the method of surrogate data might lead us to a misunderstanding. In real data analysis, there often exists a case where we must analyze quantized time series to investigate heuristic laws. Here, we describe how a real financial time series was used in order to confirm the validity of the proposed AAFT surrogate algorithms.

As the real financial time series, we used middle price movements $M(t)$, provided as tick data, between the U.S. dollar and the Swiss franc observed in the real interbank market [11], as shown in Fig. 5. In the present paper, we denote ticks by t as well as time. The tick data had $281,634 \simeq 2^{18}$ points, but the number of state values was only 369 (types), which corresponds to 8.53 bits. From the analysis, E for the AAFT, P-AAFT, and A-AAFT algorithms had the values 3.88, 0.33, and 0.31, as shown in Fig. 6. The results indicated that the P-AAFT and A-AAFT algorithms are effective for the analysis of real data. Moreover, we confirmed that the AAFT surrogate had considerably larger powers in lower frequency regions than the original data. Such discrepancies lead to a false rejection.

Then, we performed surrogate tests of the financial time series $M(t)$ with three surrogate algorithms. As a statistic for surrogate tests, we used the prediction error, described in

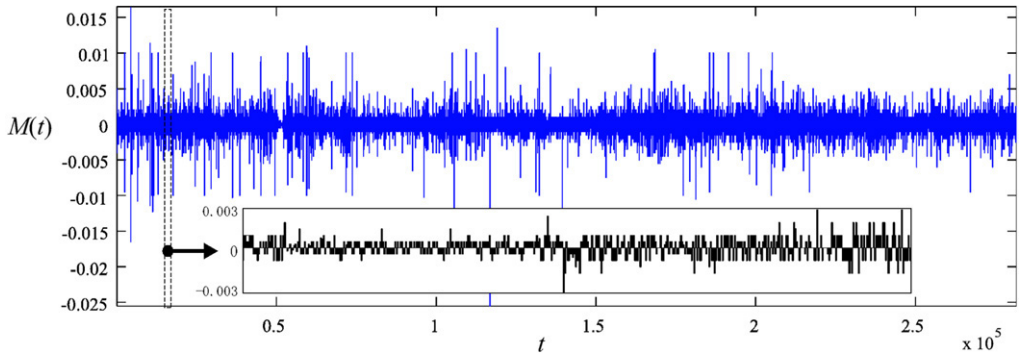


Fig. 5. The main figure shows increases and decreases of middle prices $M(t)$ of the U.S. dollar versus the Swiss franc, whose length is 281 634 [11]. The embedded figure is an enlargement of the tick series, and the horizontal indices are from $t = 19\,500$ to $t = 20\,000$.

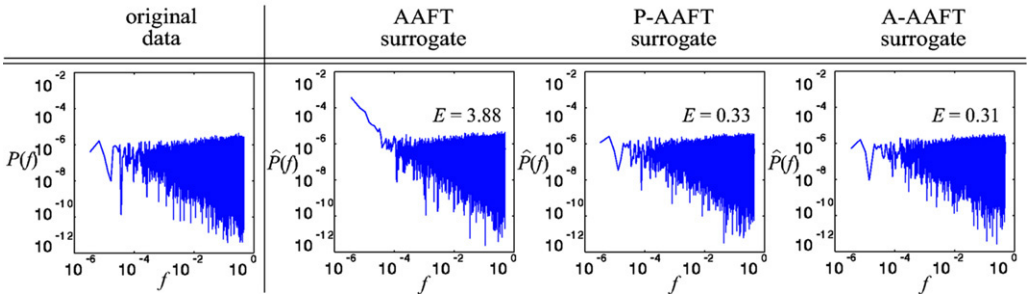
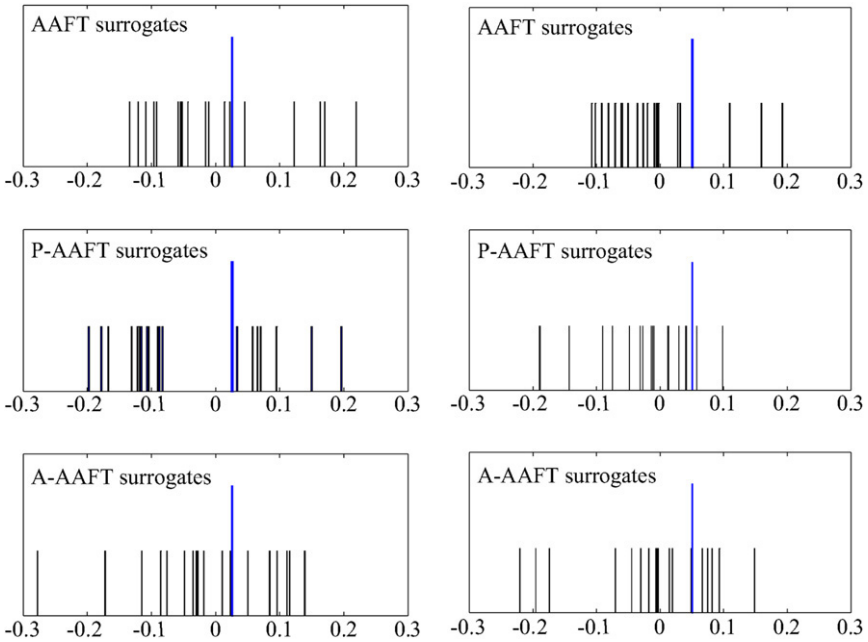


Fig. 6. The same as Fig. 2, but for the middle price movements $M(t)$. The values E shown in each figure are the relative discrepancies in Eq. (1) between two power spectra: of the original data and of the surrogate data.



(a) $D = 5$. (b) $D = 10$.

Fig. 7. The results of the surrogate test for the middle price movements $M(t)$ obtained by using AAFT surrogates (top), P-AAFT surrogates (middle), and A-AAFT surrogates (bottom). The horizontal axis of each figure means the correlation coefficient between a predicted series and the true series. Short black bars show the prediction errors of the surrogate data and long blue bars show the prediction errors of the original data. The embedding dimensions for prediction D are (a) 5 and (b) 10. No surrogate test rejects the null hypothesis. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Section 4.1, with $D = 5$ or $D = 10$. The results are shown in Fig. 7. The differences among the three surrogate tests were not confirmed; no surrogate test rejects the null hypothesis. However, since the ordinary AAFT surrogate cannot follow

the null hypothesis from the viewpoint of the extended null hypothesis introduced in the present paper, the introduction of the surrogate tests is indispensable. In other words, we must use the P-AAFT and/or A-AAFT algorithms for a reliable surrogate test even if the results derived are the same.

5. Conclusions

In the present paper, we showed the disadvantages of using the surrogate data method for coarsely quantized data and proposed two improved AAFT algorithms: the P-AAFT and A-AAFT algorithms. Then, we confirmed the validity of the two improved algorithms. The A-AAFT algorithm is better than the P-AAFT algorithm from the viewpoint of the discrepancy of the power spectrum from the original time series one. These algorithms were useful for avoiding spurious low frequency components of the AAFT surrogates. Moreover, from the results of the surrogate tests obtained by using these three AAFT algorithms, we were able to confirm that the proposed algorithms can reduce false rejections and can provide us with a more reliable surrogate test.

It must be emphasized that the proposed algorithms did not make the performance poorer if the observed time series had sufficient resolution. Of course, since the problem of the rank order of the same state value does not exist, using P-AAFT and A-AAFT algorithms has little effect. That is, $\omega(t)$, $\omega_P(t)$, and $\omega_A(t)$ are almost same. However, because the proposed algorithms are very simple, even if the data for analysis are well behaved, we recommend using the proposed algorithm for improving the reliability of the analysis.

Acknowledgement

We would like to thank the reviewers for their valuable comments and discussions. The research of TI was partially supported by Grant-in-Aids for Scientific Research (C) (Nos 13831002 and 17500136) from Japan Society for the Promotion of Science.

References

[1] P.J. Brockwell, R.A. Davis, Introduction to Time Series and Forecasting, Springer-Verlag, 1996.
[2] T. Ikeguchi, T. Yamada, M. Komuro, in: K. Aihara (Ed.), Fundamentals and Applications of Chaotic Time series Analysis, Sangyo-Tosho Ltd., 2000 (in Japanese).
[3] T. Schreibor, A. Schmitz, Surrogate time series, Physica D 142 (2000) 346–382.
[4] R.N. Mantegna, H.E. Stanley (Eds.), An Introduction of Econophysics: Correlations and Complexity in Finance, Cambridge University Press, 2000.
[5] M. Small, C.K. Tse, Determinism in financial time series, Stud. Nonlinear Dyn. Econom. 7 (3) (2003). Art. 5.
[6] J. Theiler, et al., Testing for nonlinearity in time series: The method of surrogate data, Physica D 58 (1992) 77–94.
[7] P. Grassberger, I. Procaccia, Measuring the strangeness of strange attractors, Physica 9D (1983) 189–208.
[8] J.P. Eckmann, et al., Liapunov exponents from time series, Phys. Rev. A 34 (6) (1986) 4971–4979.
[9] T. Chang, et al., Tests for nonlinearity time series, Chaos 5 (1) (1995) 118–126.
[10] T. Schreiber, A. Schmitz, Improved surrogate data for nonlinearity tests, Phys. Rev. Lett. 77 (1996) 635–638.
[11] A.S. Weigend, N.A. Gershenfeld (Eds.), Time Series Prediction, Addison-Wesley, 1993.