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Physica A 323 (2003) 591 – 600
PHYSICA A
www.elsevier.com/locate/physa

Multivariable nonlinear analysis of foreign exchange rates

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Received 15 January 2003

Abstract

We analyze the multivariable time series of foreign exchange rates. These are price movements that have often been analyzed, and dealing time intervals and spreads between bid and ask prices. Considering dealing time intervals as event timing such as neurons' firings, we use raster plots (RPs) and peri-stimulus time histograms (PSTHs) which are popular methods in the field of neurophysiology. Introducing special processings to obtaining RPs and PSTHs time histograms for analyzing exchange rates time series, we discover that there exists dynamical interaction among three variables. We also find that adopting multivariables leads to improvements of prediction accuracy.

PACS: 02.50.Sk; 05.45.Tp; 05.90.+m

Keywords: Econophysics; Exchange market; Spreads; Dealing time intervals; Raster plot; PSTH

1. Introduction

It is widely acknowledged that the market price data, e.g., stock prices and foreign exchange rates, often exhibit very complex behavior and that it is very difficult to predict their movement accurately. In order to make a good model of such financial indices and predict their complex behavior, it is essential to find which variables affect to the price movements. In the present paper, in order to solve this issue, we analyze

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0378-4371/03/$ - see front matter
doi:10.1016/S0378-4371(03)00052-9

the dealing time intervals and the spreads, the difference of bid and ask prices, of foreign exchange rates.

In the conventional studies of predicting financial data, these variables have not been often used. The reason is that the dealing time intervals have been considered to be noisy and to wonder around daily trends of business hours (that is, they are independent of the mechanism of market). However, it is very natural to anticipate that dealers' decisions could be reflected not only by a history of price movement itself but also the history of dealing time intervals and spreads. For example, Takayasu et al. [1] showed that dealing time intervals can be described by a nonstationary Poisson process in which an average value varies depending on last several minutes. This result suggests that it is a natural idea that dealers decide dealing timings on the basis of dealing data for the last several minutes. Thus, the variable of dealing timings is not independent of the mechanism of market, and it is very essential to introduce this variable for making a good model of its mechanism.

We also introduce a new variable, that is, a spread. In foreign exchange market, since there is the rule of "Two Way Quotation" that a bank must quote both bid and ask prices to another bank simultaneously, there exists a spread between bid and ask prices by every deal. It is very natural that these prices reflect the balance of demands and supplies, dealers' mind and the mood of a market which influence price movements. Namely, the variable of spreads is useful to understand the mechanism of market. In addition, we see that the spread follows a power law in our previous research [2]. Stanley, et al. also showed that the price movements have a non-gaussian distribution and also follows the power law [3]. Since the spread shows a similar statistical property to the price movements, we highly expect that the spread has relation to the mechanism of market as well as dealing time intervals.

2. The data for analysis

In the present paper, we use the time series of tick data between the US dollar and the Swiss franc observed in the interbank market [4]. The tick data is recorded from January 1986 to April 1991 (total 1322 days), and the total number of the data points is 282,956. Usually, tick data has the following intrinsic aspects. There is a sort of discontinuity on dairy tick data, because the bank closes at nights, weekends and holidays. Then the first and the last dealing times are different from each other day by day. The dealing time is recorded at actual timings when every deal occurs. We denote each middle price, namely a mean value of bid and ask prices, by P and their temporal differences by ΔP (dollar), respectively. We also denote the spread, the difference between bid and ask prices, by S (dollar) and the dealing time interval time by τ (s). Then, we use the time series data of $P(n)$, $\Delta P(n)$ $(=P(n+1)-P(n))$, $S(n)$ and $\tau(n)$ shown in Fig. 1. The temporal index n indicates a discrete time and increases by one whenever each dealing occurs.

We analyze the interaction of three variables by drawing raster plots (RPs) and peri-stimulus time histograms (PSTHs) [5–7]. These methods are often utilized for analyzing spike data in the field of neurophysiology, and are good schemes to express

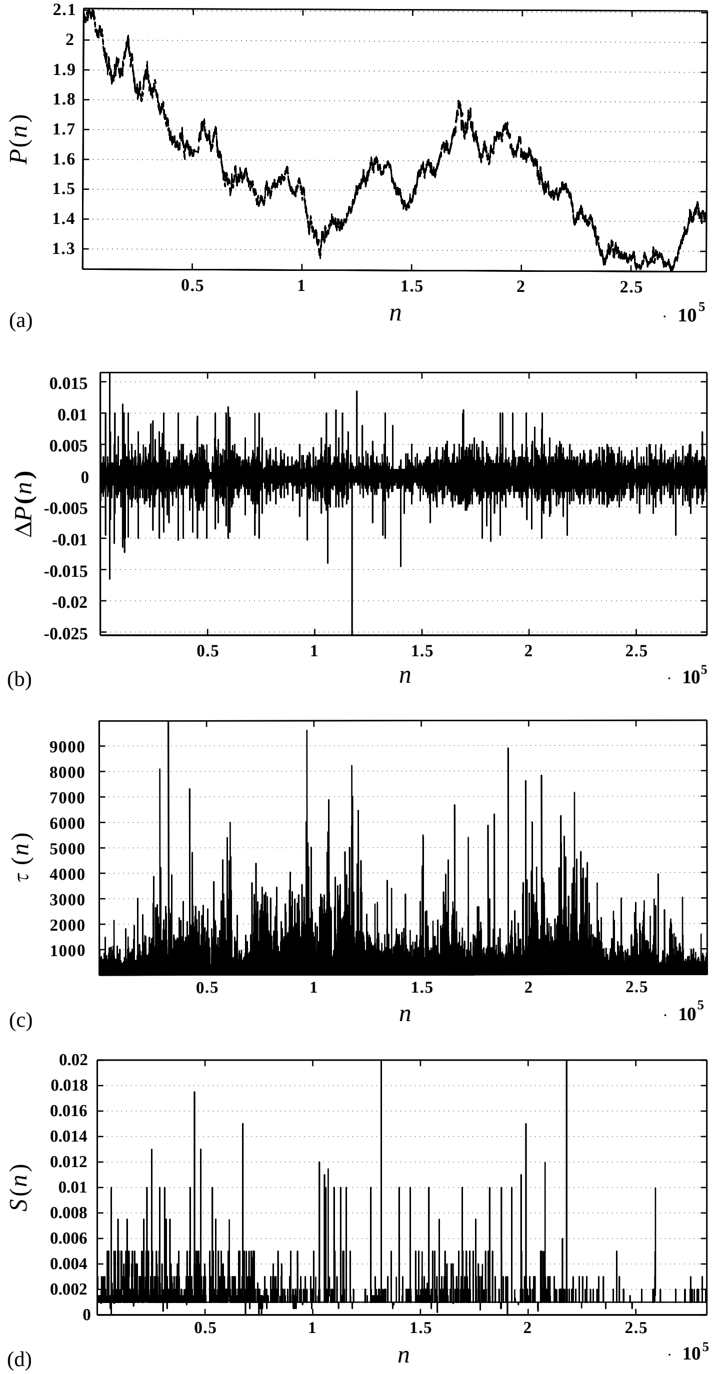

Fig. 1. Real time series of (a) $P(n)$, (b) $\Delta P(n)$, (c) $\tau(n)$ and (d) $S(n)$.

spike timings visually. In the case of analyzing neural spikes, every observed spike is plotted in horizontal lines with external stimuli. However, for analyzing financial indices it is not easy to consider the existence of external stimuli. Then we use the following way in order to apply these techniques to analyze interbank data.

For drawing RPs, we consider each day as each trial, and occurrences of actual dealings as spikes. Then, since the case in which spreads become very large is not so often observed, we assume that the particular time at which spreads become very large is equal to the time at which external stimuli are applied in order to investigate the alteration of market's behavior by external stimuli. For evaluating PSTHs, we have to consider how to treat the variation of opening and closing time in each day. In the ith temporal bin ($i=1,2,\dots,B$), deals are denoted by $s_i(k)$ ($k=1,\dots,N_i$). When there are d_i days in which the start of dealing is later and the finish of dealing is earlier than the median of the corresponding temporal bins, the average of the dairy dealing in the ith temporal bin is calculated by

$$H_i = \frac{N_i}{D - d_i} , \tag{1}$$

where D is the number of total days. Moreover, in order to examine an ensemble behavior of the movement of middle prices, the difference of the middle price in the dealing $s_i(k)$ is used for calculating the following histogram,

$$h_i = \frac{1}{N_i} \sum_{k=1}^{N_i} |\Delta P_{s_i(k)}| , \tag{2}$$

where $\Delta P_{s_i(k)} = P_{s_i(k)} - P_{r_i(k)}$, and $P_{r_i(k)}$ is a one-step previous price of $P_{s_i(k)}$. Then, we divide the total dealing length from 9:00 a.m. to 5:00 p.m. into four sections (each has 2 h long) and classify all tick data into each section on the basis of the dealing time.

We calculate RPs and PSTHs by the following two methods (i) and (ii) and compare both of these results.

(i) We randomly select 15 dealings, in which the actual dealings occur nearly at the median of each section, and temporarily identify their dealing time as the time of external stimuli. Then, their deals are placed on the vertical line at $t=0$ on the horizontal axis in RPs. Here, t is continuous time.
(ii) We select top 15 dealing data sets whose spreads become much larger from each temporal section. Stimuli are considered to be the same as those in (i).

If the movement of spreads has no relation to the dealer's action, no significant difference appears between the results obtained by the above two methods.

Figs. 2 and 3 show the results obtained by the above analysis. The temporal section is from 11:00 a.m. to 1:00 p.m, $B=40$ (the temporal bin is about 1000 (s)) and $D=15$. In Fig. 2, the histogram H_i shows a kind of temporal rhythm (a shape of the letter "M") and the histogram h_i is almost flat (no particular rhythm). The M type rhythm shows a daily trend of the dealing time intervals which depend on the number of dealers who participate in the market. Since there are few dealers in early morning, lunch time and evening, the dealing time intervals tend to become larger in such time

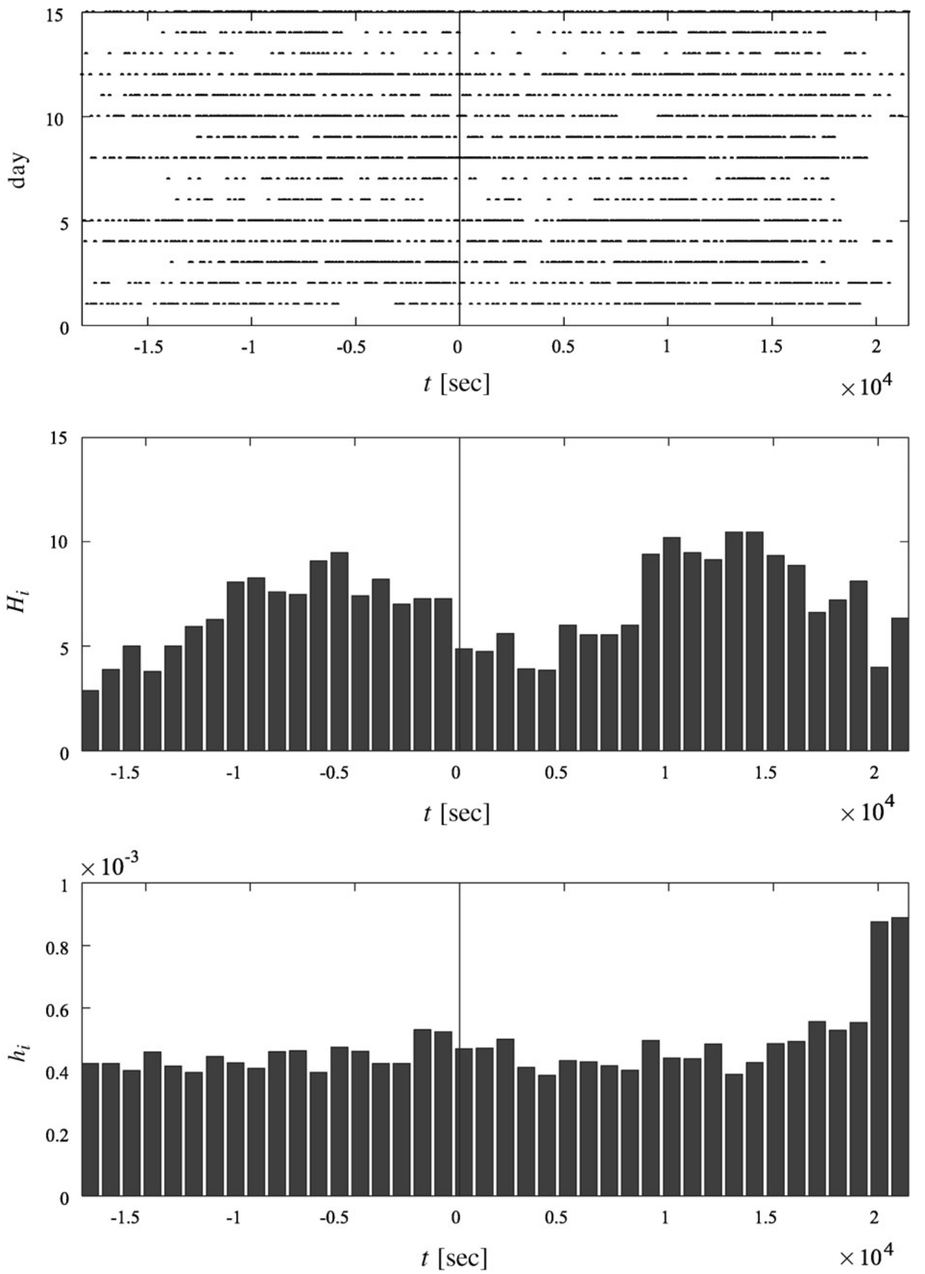

Fig. 2. The results by method (i).

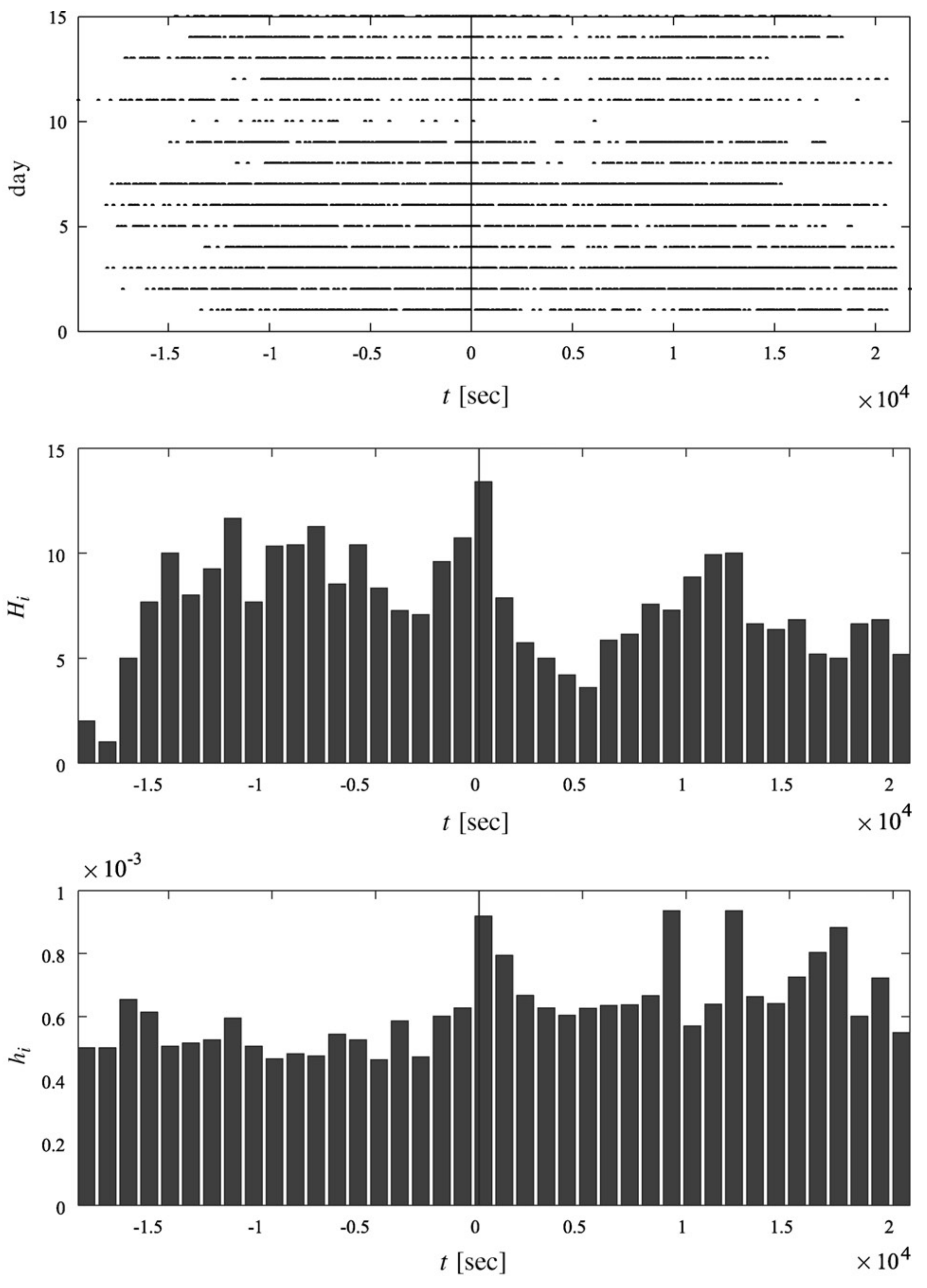

Fig. 3. The results by method (ii).

regions. Then, H_i becomes shorter and the M type rhythm appears. On the other hand, we find, from Fig. 3, that many dealings appear in temporal bins near $t = 0$ and the large amount of dealings destroys the rhythm of the shape "M", and the dealing time interval becomes shorter. Moreover, the histogram h_i also shows that the movement of middle prices has a peak in the temporal bins at $t = 0$, which means that the expansion of the spreads makes the movement of middle prices larger.

The term *expansion of the spread* means that the bid and ask prices separate from the middle price, which indicates that dealers try to buy with lower prices and to sell with higher prices. It is very interesting that in spite of such bull quotations, this dealing time interval becomes shorter. Such tendencies are also observed if we use other three sections (9:00–11:00 a.m., 1:00–3:00 p.m. and 3:00–5:00 p.m.) as the central time. Namely, it is shown that the expansion of the spreads often makes the dealing time intervals shorter and the movement of prices larger, and it supports our conjecture that three variables dynamically interact with each other.

3. Nonlinear modeling with three variables

If there is a dynamical relationship between $|\Delta P|$, τ and S, adopting three variables would improve prediction performance. Since the movement of spreads is much smaller than the others, we have to use a large amount of the learning data. Here, we have to treat again the issue that there is a discontinuity of dairy data. Namely, simple connection of each dairy data might lead us to spurious results, because the dealers cannot deal from closing time to opening time of the next day, and because the flow of information and the change of dealers' minds in the closed period of the exchange market are both ignored. To solve this issue, when we adopt a local linear approximation method described by the following equation [8] in the case of predicting the future of $v(n)$,

$$\hat{v}(n+1) = \frac{\sum_{j=1}^{M} \exp(-l_j) v(k_j + 1)}{\sum_{j=1}^{M} \exp(-l_j)} , \tag{3}$$

$$l_j = |v(n) - v(k_j)| , \tag{4}$$

where $v(k_j)$ is the jth nearest neighbor of $v(n)$, $M = m + 1$ is the number of near neighbors, m denotes the embedding dimension [9] and $\hat{v}(n+1)$ is a predicted point, we use a constraint for selecting nearest neighbors. Although there is no dealing during the closed period of the exchange market, we assume that the information about the exchange market and dealers' minds change continuously. Thus, there exists a sort of a possible attractor with underlying deterministic dynamics. In Fig. 4, dotted lines denote trajectories of the attractor, and the points on this attractor are unknown. Moreover, the last points of each day are not suitable as a near neighbor, because it does not have any corresponding future values. Namely, we use Eqs. (3) and (4) so that we do not select open circles in Fig. 4 as nearest neighbors.

With the modified neighbor selection method, we predict the movement of $|\Delta P|$ for the 50 days, which we randomly select from total 1322 days data. For learning data we

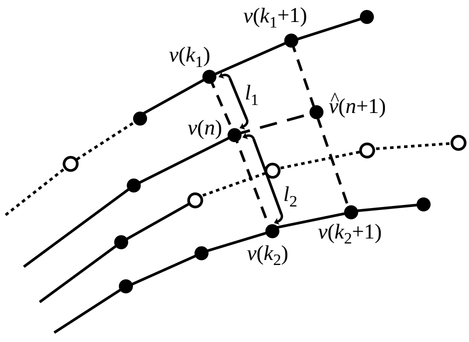

Fig. 4. Schematic representation of the proposed method. Solid lines are the attractor of observed dealing data. Dotted lines are the unknown attractor.

use all the data which have early temporal indices smaller then n. In order to compare prediction results, we reconstruct the attractor ($m = 12$) in the following four ways.

$$v_1(n) = \{|\Delta P(n-11)|, \ldots, |\Delta P(n-1)|, |\Delta P(n)|\} , \quad (5)$$

$$v_2(n) = \{|\Delta P(n-5)|, \ldots, |\Delta P(n)|, \tau(n-5), \ldots, \tau(n)\} , \quad (6)$$

$$v_3(n) = \{|\Delta P(n-5)|, \ldots, |\Delta P(n)|, S(n-5), \ldots, S(n)\} , \quad (7)$$

$$v_4(n) = \{|\Delta P(n-3)|, \ldots, |\Delta P(n)|, \tau(n-3), \ldots, \tau(n), S(n-3), \ldots, S(n)\} . \quad (8)$$

Since each variable has a different scale, we normalize each datum in prediction, and we rescale predicted data. For estimating prediction performance, we use the root mean square errors E [10] and the signature errors R [10]:

$$E = \frac{\sqrt{\sum_{n=1}^{L} (z(n) - \hat{z}(n))^2}}{\sqrt{\sum_{n=1}^{L} (z(n) - \bar{z})^2}} , \quad (9)$$

$$R(\%) = 100 - \frac{\sum_{n=1}^{L} I(Z(n)\hat{Z}(n))}{L} \times 100 , \quad (10)$$

where z is the true time series, $\hat{z}$ is the predicted time series, $\bar{z}$ is the mean of real time series, L is the number of data points and $I(n)$ is a step function. Here, $Z(n) = z(n) - z(n-1)$ and $\hat{Z}(n) = \hat{z}(n) - z(n-1)$. Thus, R shows the prediction performance concerning whether $|\Delta P|$ becomes large or small. Table 1 shows the results of prediction. We calculate E and R with the prediction results for 50 days and also calculate the mean value of E and R, μ_E and μ_R, and the variance of E and R, σ_E^2 and σ_R^2. Then, we denote B_E and B_R as the rate that the prediction performance of each day becomes better in the case of using multivariables (Eqs. (6)–(8)) more than using one variable (Eq. (5)). In Table 1, we can confirm that the prediction performance measured by E and R is much improved if we use more variables for reconstruction attractors.

Table 1
Results of prediction

Eq.	μ_E	σ_E^2	B_E	μ_R	σ_R^2	B_R
(5)	1.0569	0.0124	—	18.2168	20.7828	—
(6)	1.0335	0.0078	52(%)	18.1473	22.6411	54(%)
(7)	1.0267	0.0084	61(%)	17.6735	14.0682	61(%)
(8)	1.0278	0.0069	61(%)	17.1838	19.6670	74(%)
(11)	1.0415	0.0034	—	42.9032	17.5400	—
(12)	1.0384	0.0024	42(%)	41.6164	16.9700	60(%)
(13)	1.0486	0.0060	46(%)	42.1057	20.4015	58(%)
(14)	1.0386	0.0031	48(%)	41.4581	15.7351	68(%)

Next, we apply the prediction algorithm to ΔP instead of $|\Delta P|$, since the prediction of ΔP is useful for us than prediction of $|\Delta P|$ in actual dealing. Reconstructed attractors are as follows:

$$v_1(n) = \{\Delta P(n-11), \ldots, \Delta P(n-1), \Delta P(n)\} , \tag{11}$$

$$v_2(n) = \{\Delta P(n-5), \ldots, \Delta P(n), \tau(n-5), \ldots, \tau(n)\} , \tag{12}$$

$$v_3(n) = \{\Delta P(n-5), \ldots, \Delta P(n), S(n-5), \ldots, S(n)\} , \tag{13}$$

$$v_4(n) = \{\Delta P(n-3), \ldots, \Delta P(n), \tau(n-3), \ldots, \tau(n), S(n-3), \ldots, S(n)\} . \tag{14}$$

In this case, we define as $Z(n) = z(n)$ and $\hat{Z}(n) = \hat{z}(n)$ in order to correctly estimate prediction accuracy for predicting ups and downs of $P(n)$. The contents of Eqs. (11)–(14) in Table 1 show these prediction results. The performance measured by E is not improved significantly. In particular, the result due to Eq. (13) is worse than the that of Eq. (11) and every B_E is under 50(%). However, the prediction performance measured by R is always more improved if we use multivariables for reconstructing attractors as well as former prediction results by Eqs. (6)–(8). If the dealing time interval $\tau(n)$ and the spread $S(n)$ are independent of price movements $|\Delta P(n)|$ and $P(n)$, prediction performance measured by E and R could be worse if we use $\tau(n)$ or $S(n)$ for prediction. Namely, this fact that prediction results become better suggests that there exists a dynamical relationship among these variables.

4. Conclusions

In the present paper, we show the existence of a dynamical interaction among the middle prices, the dealing time intervals and the spreads from the viewpoint of ensemble behavior in each day. We also use multivariables for a reconstructing attractor and modify the conventional local linear approximation method in order to treat discontinuity of dairy data. As a result, our scheme with multivariable reconstruction improves all signature errors and root mean square errors, which means that our scheme can predict the price movements more precisely. These prediction results also show the existence of interaction among these variables.

It is a very important future problem to understand the reason why these three variables interact each other and to investigate the degree of its interaction. We will discuss this issue by modeling foreign exchange markets elsewhere [2].

The research of TI was partially supported by Grant-in-Aids for Scientific Research (C) (No. 13831002) from Japan Society for the Promotion of Science, and for Scientific Research on Priority Areas (No. 14016002) from Ministry of Education, Culture, Sports, Science and Technology.

References

[1] H. Takayasu, M. Takayasu (Eds.), Econophysics-Toward Scientific Reconstruction of Economy, Nihon Keizai Shimbun, 2001 (in Japanese).
[2] T. Suzuki, T. Ikeguchi, M. Suzuki, Modeling on complex behavior of interbank exchange markets, 2003, submitted for publication.
[3] R.N. Mantegna, H.E. Stanley, Scaling behavior in the dynamics of an economic index, Nature 376 (1995) 46–49.
[4] A.S. Weigend, N.A. Gershenfeld (Eds.), Time Series Prediction, Addison-Wesley, Reading, MA, 1993.
[5] G.L. Gerstein, M.J. Bloom, I.E. Espinosa, S. Evanczuk, M.R. Turner, Design of a laboratory for multineuron studies, IEEE Trans. Syst. Man Cybern. SMC-13 (1983) 668–676.
[6] J. Krueger, Simultaneous individual recordings from many cerebral neurons: techniques and results, Rev. Physiol. Biochem. Pharmac. 98 (1983) 177–233.
[7] C.M. Gray, P. Koenig, A.K. Engel, W. Singer, Oscillatory responses in cat visual cortex exhibit inter-columnar synchronization which reflects global stimulus properties, Nature 338 (1989) 334–337.
[8] G. Sugihara, R.M. May, Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series, Nature 344 (1990).
[9] F. Takens, Detecting strange attractors in turbulence, in: D.A. Rand, L.S. Young (Eds.), Dynamical System and Turbulence, Lecture Notes in Mathematics, Vol. 898, Springer, Berlin, 1981, pp. 366–381.
[10] T. Ikeguchi, K. Aihara, Prediction of chaotic time series with noise, IEICE Transactions on Fundamentals of Electronics, Commun. Comput. Sci. E78-A (10) (1995) 1291–1297.