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A model of complex behavior of interbank exchange markets

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Abstract

In the present paper, we analyze the complex interaction among three macroscopic variables, dealing time intervals, spreads between ask and bid prices and price movements, observed in actual interbank exchange markets. For this analysis, we propose a new model of interbank exchange dealings as a statistical system integrated by many dealers' actions with the methods of statistical physics. For evaluating the plausibility of our model, we compare outputs from the proposed model with the real data by reconstructing a state space with the above three variables, observing ensemble behavior in each day and estimating statistical properties. As a result, we can confirm that our model is plausible, and we perform the above analysis with our model from the viewpoint of statistical physics.

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1. Introduction

In recent years, several pioneering studies [1–3] have discussed complex behavior of financial phenomenon from the viewpoint of statistical physics. Before these studies [1–3] have been published, financial phenomena were not treated as physics, since financial phenomena were often considered to be disturbed by dealers' mind and exter-

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nal information. However, it has been clarified that global dealers' minds and actions follow statistical and objective laws [1–3] in an open market even if many dealers trade freely with sharing almost the same information. Namely, it is very useful to apply the concepts and methods of statistical physics to financial phenomena. Then, another interesting studies have been reported [4–12], which are often referred as Econophysics [13].

However, the variety of discussing on financial activities is not wide yet. In particular, very few discussions have been made on the decision mechanism of bid and ask prices of dealings in interbank exchange markets, even in the field of economics [14]. One of its reasons is that the modeling of this mechanism depends on dealing processes, say in interbank exchange dealings and in stock dealings. While a dealer quotes ask and bid price simultaneously in the former, ask and bid price are quoted by different dealers in the latter.

Thus if we try to find a general model, which is able to treat both interbank exchange markets and stock markets, we have to neglect such variables as ask and bid prices in a model. However, if we focus on making only a model of an interbank exchange market, we may use the spread between bid and ask prices. Since these spreads provide important information of dealers' mind of earnings and hedging risks, the movement of the spreads could be one of the essential factors for modeling price movements.

In the present paper, we propose a model of interbank exchange dealing by using not only price movements but also the spreads and the dealing time intervals. In our previous study [15], using real data of the interbank exchange market between the Swiss franc and the US dollar, we have already analyzed the interaction among the price movements, the dealing time intervals and the spreads of real data, and we have discovered that when the spread becomes larger, the dealing time interval becomes shorter and the movement of price becomes larger [15]. Since the expansion of the spread means that ask and bid prices are separated from the middle price, it is natural to consider that the dealer tries to sell at higher prices and to buy at lower prices. Such a bull quotation is anticipated to lead to the situation that dealing time intervals become longer when the spread becomes larger, since it is not so easy to find a dealing partner. However, our analyses [15] show a completely opposite tendency. One of the motivations of the present paper is to construct a novel model which could explain this remarkable tendency from the viewpoint of statistical physics.

The present paper is organized as follows. In Section 2.1, we model the process of deciding bid and ask prices in the interbank exchange market. In our model, we consider the distribution of possible future prices by introducing a geometric Brownian motion [16,17]. Then, we extend it to the distribution from which each dealer expects future prices. In addition, the variance of the distribution corresponds to a fluctuation term since dealers' action is considered as behavior of particles. Next, we formulate the deciding mechanism of the spread by each dealer. In economics, there is a fundamental concept that dealers always act most rationally. Namely, we can naturally consider that each dealer decides the best spread so that an expected utility is maximized. However, research results of Refs. [1–13] have not used this concept, since it is not so easy to treat or describe such dealers' mind in a quantitative way. Then, if we consider the reciprocal of the expected utility as a potential energy in physics, the most stable state, that is the best spread, is realized by maximizing the expected utility depending on past information. Thus, in our model, we treat dealers' mind as a matter with a universal property by assuming the natural property that the dealers always act most rationally.

In Section 2.2, we make a model for a process of dealing execution in order to decide dealing time intervals. Here, all dealers are assumed to take part in the model market with the formulae proposed in Section 2.1 and find a dealing partner randomly. When a dealer succeeds in getting a deal with the dealing partner, a dealing time interval is decided. In Section 2.3, we introduce Auto Regressive Conditional Heteroscedasticity (ARCH) process [18] in order to use it as surrogate data of the price movements. In Section 3, we estimate several statistical properties on the real data in order to show that the outputs from the proposed model have similar stochastic properties. In Section 4, we show that the same property appears in the data obtained by the proposed model and we also discuss the reason why it appears in the proposed model and the real data from the viewpoint of statistical physics.

2. Proposition of our model

We assume that dealers' quotations of bid and ask prices are only determined by the past information. However, in our model we try to consider only the past movement of middle prices between ask and bid prices. Namely, we neglect here the volume of dealings, not only for the sake of simplicity but also for discussing an essential aspect of complex behavior of price movements.

2.1. Dealers' action

In this section, we consider a model of the decision mechanism for the spread on the basis of real dealing action by dealers. First, we formulate the possible future prices by introducing a geometric Brownian motion [16,17]:

$$\mathrm{d}P(n+\mathrm{d}n) = \mu(n)P(n)\,\mathrm{d}n + \sigma(n)P(n)\,\mathrm{d}W \;, \tag{1}$$

where n is a present temporal index, $P(n)$ is a middle price, $\mathrm{d}W \sim N(0, \mathrm{d}n)$ and $N(0, \mathrm{d}n)$ is a standard normal distribution. Next, we define two coefficients $\mu(n)$ and $\sigma(n)$ in Eq. (1) as follows. Let us assume to have the past information of middle prices, which are denoted by $P(\alpha)$, $(\alpha = 1, 2, \ldots, n)$. Here α is the temporal index, that is, $\alpha = 1$ indicates that it is the first term and $P(1)$ is the first information. Since the index n increases one by one whenever each dealing occurs, we consider that $\mathrm{d}n = 1$. Consequently we have $\mathrm{d}P(n+1) = P(n+1) - P(n)$. Then, we define $\mu(n)$ and $\sigma(n)$ by the mean and standard deviations of the movements $\mathrm{d}P(w)/P(w-1)$ (for $w = n - p + 1, \ldots, n$), which means the return rates of middle prices during the last p terms.

Now, let us introduce two new variables $\bar{P}$ and γ to modify Eq. (1) as

$$P(n+1) = \bar{P}(n+1) + \gamma(n+1)\,\mathrm{d}W \;, \tag{2}$$

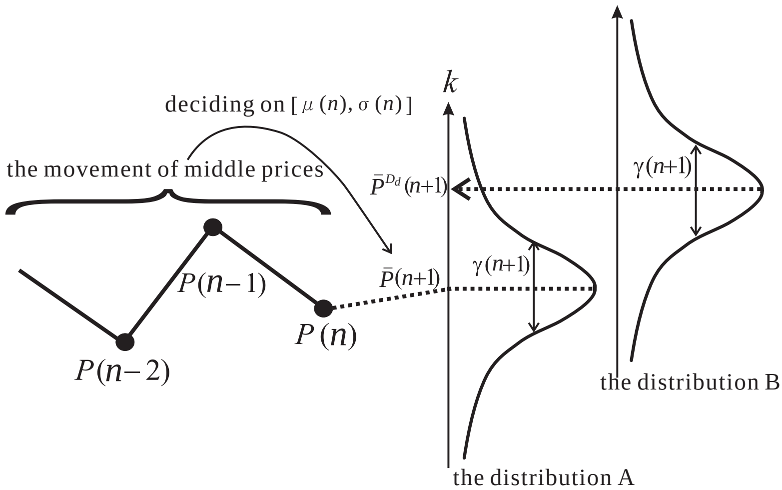

Fig. 1. As past information, we only utilize the movement of past middle prices P. The distribution A offers a possibility of future prices at the $(n+1)$-th dealing as shown in Eq. (2). The distribution B is an expected distribution of a future price by a dealer D_d whose expected value of the future price is $\bar{P}^{D_d}(n+1)$ as shown in Eq. (7). The standard deviations of these distributions are described as $\gamma(n+1)$.

where

$$\bar{P}(n+1) = \mu(n)P(n) + P(n) = (\mu(n)+1)P(n) , \tag{3}$$

which describes the effect of preserving the trend of price movements, and

$$\gamma(n+1) = \sigma(n)P(n) , \tag{4}$$

which describes the effect of maintaining the intensity of price movements. Eq. (2) indicates that there exists a distribution of possible future prices as shown in Fig. 1. We call it the distribution A. The variable $\bar{P}(n+1)$ is the mean value of the distribution A and $\gamma(n+1)$ is the standard deviation of the distribution A.

Dividing both hands sides by $P(n)$, Eq. (1) is followed by

$$\frac{\mathrm{d}P(n+1)}{P(n)} = \mu(n) + \sigma(n)\,\mathrm{d}W , \tag{5}$$

which means that the return rates follow a Brownian motion. Moreover, since

$$\begin{aligned}
\mathrm{d}\log P(n+1) &= \log\left(\frac{P(n+1)}{P(n)}\right), \\
&= \log\left(\frac{P(n)+\mathrm{d}P(n+1)}{P(n)}\right), \\
&= \log\left(1+\frac{\mathrm{d}P(n+1)}{P(n)}\right), \\
&\simeq \frac{\mathrm{d}P(n+1)}{P(n)} ,
\end{aligned} \tag{6}$$

the return rate means the movement of logarithmic prices. It should be noted that Eq. (6) is the first-order approximation of the Taylor series.

Now, we generalize dealers' actions to model the deciding mechanism of the spreads between bid and ask prices. In Eq. (2), a best prediction value for the term $n+1$ surely exists and it is $\bar{P}(n+1)$, because it takes the highest probability (of course it is unknown). However, there exists a wide variety of expected prediction values $\bar{P}^{\boldsymbol{D}}(n+1)$, which depend on dealers, $\boldsymbol{D}=\{D_i, i=1,2,\ldots,I\}$, where D_i denotes the ith dealer. Thus, in order to model the decision mechanism of prices, we consider these expected best predictions $\bar{P}^{\boldsymbol{D}}(n+1)$ by the dealers $\boldsymbol{D}$ as follows. Since a future price will be decided by the predicted price of a dealer who will be able to get a real deal, possible future prices correspond to the predicted future prices by the dealers $\boldsymbol{D}$. Thus, the distribution of $\bar{P}^{\boldsymbol{D}}(n+1)$ should be the same as the distribution A (the elements of the distribution A are both possible future prices $P(n+1)$ and the future prices expected by other dealers $\boldsymbol{D}$).

Now, let us consider dealing actions by dealers included in distribution A. From Eq. (2), a distribution of future prices expected by the dth dealer (D_d), $P^{D_d}(n+1)$, is described by

$$P^{D_d}(n+1)=\bar{P}^{D_d}(n+1)+\gamma(n+1)\,\mathrm{d}W\ , \tag{7}$$

where $\bar{P}^{D_d}(n+1)$ is the mean value of the distribution of prediction values by the dealer D_d and $\gamma(n+1)$ is the standard deviation of the distribution, which is a risk for predicting future price and which is denoted as the same as Eq. (2) for simplicity. Namely, Eq. (7) explains the existence of the distribution B as shown in Fig. 1.

Next, in order to obtain an expected return of one step future at the term n, we try to calculate expected values of the gain $G^{D_d}(n+1)$ and the loss $L^{D_d}(n+1)$ at the term $n+1$. It should be noted that $G^{D_d}(n+1)$ and $L^{D_d}(n+1)$ depend on bid and ask prices quoted by the dealer D_d. We consider K sample variables of the distribution B of Eq. (7), $x_k^{D_d}(n+1)$ $(k=1,2,\ldots,K)$. Since it is very natural for dealers to try to sell at higher prices and to buy at lower prices, we denote the ath "largest" variable of them as an ask price $x_a^{D_d}(n+1)$ and the bth "smallest" variable as a bid price $x_b^{D_d}(n+1)$. Here $a=b$, since the center of the distribution B is the predicted future "middle" price. Moreover, when the actual future price $P(n+1)$ becomes $x_k^{D_d}(n+1)$, $x_a^{D_d}(n+1)-x_k^{D_d}(n+1)$ or $x_k^{D_d}(n+1)-x_b^{D_d}(n+1)$ are returns of the dealer D_d.

Next, the dealer D_d considers that if $P(n+1)=x_k^{D_d}(n+1)$, there exists a prediction error of $\bar{P}^{D_d}(n+1)-x_k^{D_d}(n+1)$ in the distribution B of Fig. 2. Since $\bar{P}^{D_d}(n+1)$ and $\bar{x}^{D_d}(n+1)$ are the mean values of the same distribution, $\bar{P}^{D_d}(n+1)=\bar{x}^{D_d}(n+1)$. Then, the prediction error is given by $\bar{x}^{D_d}(n+1)-x_k^{D_d}(n+1)$. Moreover, the dealer D_d considers that predicted future prices $\bar{P}^{\boldsymbol{D}'}(n+1)$ by dealers other than the dealer D_d, $\boldsymbol{D}'=\{D_i,\ i=1,\ldots,I, i\neq d\}$, follows the distribution C shown in Fig. 2. The variable $\bar{P}^{\boldsymbol{D}'}(n+1)$ is described by

$$\bar{P}^{\boldsymbol{D}'}(n+1)=x_k^{D_d}(n+1)+\gamma(n+1)\,\mathrm{d}W\ . \tag{8}$$

The meaning of Eq. (8) is that the predicted future prices by dealers $\boldsymbol{D}'$ distribute around the actual future price $x_k^{D_d}(n+1)$, or $P(n+1)$, which is a very natural

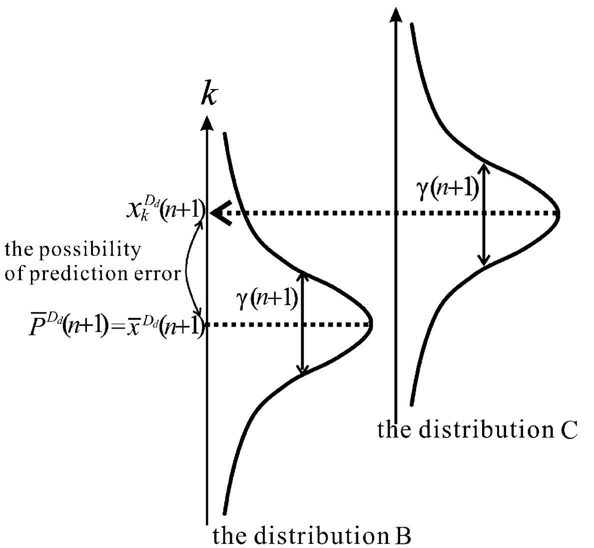

Fig. 2. A dealer D_d predicts a future price that drops in distribution B. The dealer D_d considers that if $P(n+1)=x_k^{D_d}(n+1)$, there is a prediction error between the expected value of the future price $\bar{P}^{D_d}(n+1)$ and the real future price $P(n+1)$. Then, the dealer also considers the expected values of the future price $\bar{P}^{\boldsymbol{D}'}(n+1)$ by other dealers $\boldsymbol{D}'$ ($\boldsymbol{D}'=\{D_i,\ i=1,\ldots,I, i\neq d\}$), which are included in the distribution C as shown in Eq. (8).

interpretation. Generally, the actual future price corresponds to the mean value of the future prices predicted by all dealers.

The natural condition that the dealer D_d gets a bid deal or an ask deal with dealers $\boldsymbol{D}'$ is given by $x_a^{D_d}(n+1)\leqslant \bar{P}^{\boldsymbol{D}'}(n+1)$ or $x_b^{D_d}(n+1)\geqslant \bar{P}^{\boldsymbol{D}'}(n+1)$. That is, the region where this condition is satisfied is defined by $f_C(\geqslant x_a^{D_d}(n+1))$ or $f_C(\leqslant x_b^{D_d}(n+1))$. Here, f_C is a probability distribution function of the distribution C of Eq. (8). The difference of the mean value between the distributions B and C corresponds to the prediction error of the dealer D_d, $\bar{x}^{D_d}(n+1)-x_k^{D_d}(n+1)$. Then, $f_C(\geqslant x_a^{D_d}(n+1))$ and $f_C(\leqslant x_b^{D_d}(n+1))$ are described by $f_B(\geqslant x_a^{D_d}(n+1)+\{\bar{x}^{D_d}(n+1)-x_k^{D_d}(n+1)\})$ and $f_B(\leqslant x_b^{D_d}(n+1)+\{\bar{x}^{D_d}(n+1)-x_k^{D_d}(n+1)\})$, respectively. Then, an expected value of the gain $G^{D_d}(n+1)$ is calculated as follows:

$$
\begin{aligned}
G^{D_d}(n+1) = \sum_{k=1}^{a} & \frac{1}{K}(x_a^{D_d}(n+1)-x_k^{D_d}(n+1)) \\
& \times f_B(\geqslant x_a^{D_d}(n+1)+\{\bar{x}^{D_d}(n+1)-x_k^{D_d}(n+1)\}) \\
& + \sum_{k=b}^{K}\frac{1}{K}(x_k^{D_d}(n+1)-x_b^{D_d}(n+1)) \\
& \times f_B(\leqslant x_b^{D_d}(n+1)+\{\bar{x}^{D_d}(n+1)-x_k^{D_d}(n+1)\})\,,
\end{aligned} \tag{9}
$$

where the first term of Eq. (9) is the expected value that the dealer D_d can get an ask dealing at a higher price than a real future price $x_k^{D_d}(n+1)$, or $P(n+1)$, and the second term of Eq. (9) is the expected value that the dealer D_d can get a bid dealing at a lower price than the real future price $x_k^{D_d}(n+1)$, or $P(n+1)$. By the symmetry of the distribution, Eq. (9) is reduced to

$$G^{D_d}(n+1) = \frac{2}{K}\sum_{k=1}^{a}\left(x_a^{D_d}(n+1) - x_k^{D_d}(n+1)\right) \times f_B(\geqslant x_a^{D_d}(n+1) + \{\bar{x}^{D_d}(n+1) - x_k^{D_d}(n+1)\}) \,. \tag{10}$$

In the same way as in Eq. (9), an expected value of the loss $L^{D_d}(n+1)$ is calculated as

$$\begin{aligned} L^{D_d}(n+1) = &\sum_{k=a}^{K}\frac{1}{K}(x_k^{D_d}(n+1) - x_a^{D_d}(n+1)) \\ &\times f_B(\geqslant x_a^{D_d}(n+1) + \{\bar{x}^{D_d}(n+1) - x_k^{D_d}(n+1)\}) \\ &+\sum_{k=1}^{b}\frac{1}{K}(x_b^{D_d}(n+1) - x_k^{D_d}(n+1)) \\ &\times f_B(\leqslant x_b^{D_d}(n+1) + \{\bar{x}^{D_d}(n+1) - x_k^{D_d}(n+1)\}) \,, \end{aligned} \tag{11}$$

where the first term is the expected value that the dealer D_d gets an ask dealing at a lower price than a real future price $x_k^{D_d}(n+1)$, or $P(n+1)$, and the second term is the expected value that the dealer D_d get a bid dealing at a higher price than a real future price $x_k^{D_d}(n+1)$, or $P(n+1)$. Eq. (11) is also reduced to the following equation by the symmetry of the distribution:

$$L^{D_d}(n+1) = \frac{2}{K}\sum_{k=a}^{K}\left(x_k^{D_d}(n+1) - x_a^{D_d}(n+1)\right) \times f_B\left(\geqslant x_a^{D_d}(n+1) + \{\bar{x}^{D_d}(n+1) - x_k^{D_d}(n+1)\}\right) \,. \tag{12}$$

Thus, an expected return $E^{D_d}(n+1)$ at the term $n+1$ is calculated as

$$\begin{aligned} E^{D_d}(n+1) &\equiv G^{D_d}(n+1) - L^{D_d}(n+1) \\ &= \frac{2}{K}\left(\sum_{k=1}^{K} f_B(\geqslant x_a^{D_d}(n+1) + \{\bar{x}^{D_d}(n+1) - x_k^{D_d}(n+1)\})x_a^{D_d}(n+1)\right. \\ &\left. -\sum_{k=1}^{K} f_B(\geqslant x_a^{D_d}(n+1) + \{\bar{x}^{D_d}(n+1) - x_k^{D_d}(n+1)\})x_k^{D_d}(n+1)\right) \end{aligned}$$

$$= \frac{2}{K}\sum_{k=1}^{K} f_B(\geqslant x_a^{D_d}(n+1) + \{\bar{x}^{D_d}(n+1) - x_k^{D_d}(n+1)\})$$

$$\times (x_a^{D_d}(n+1) - x_k^{D_d}(n+1)) . \tag{13}$$

It is very natural to assume that dealers will decide bid and ask prices by maximizing $E^{D_d}(n+1)$. This maximization corresponds to the fact that it is most stable in potential energy where dealers act most rationally as introduced in Section 1. Then, we denote "decided ask prices" by $\dot{x}_a^{D_d}(n+1)$ and "decided bid prices" by $\dot{x}_b^{D_d}(n+1)$, and also we denote the maximized value of $E^{D_d}(n+1)$ by $\dot{E}^{D_d}(n+1)$. Moreover, using $\dot{x}_a^{D_d}(n+1)$ and $\dot{x}_b^{D_d}(n+1)$, a decided spread is evaluated as

$$\dot{S}^{D_d}(n+1) = \dot{x}_a^{D_d}(n+1) - \dot{x}_b^{D_d}(n+1) . \tag{14}$$

In the above discussion, we have derived how the dealer D_d decides spreads. Even if we use another dealer D_i instead of D_d, we can discuss in the same way using Eqs. (7)–(14). Namely, $\dot{E}^{D_d}(n+1)$ and $\dot{S}^{D_d}(n+1)$ do not depend on D_i $(i=1,\ldots,d,\ldots,I)$ but they depend only on the temporal movement of the standard deviation $\gamma(n+1)$. Thus, we can denote them as $\dot{E}(n+1)$ and $\dot{S}(n+1)$. Then, dealer's quotations of ask and bid prices are

$$P_a^{D_i}(n+1) = \bar{P}^{D_i}(n+1) + \dot{S}(n+1)/2 , \tag{15}$$

$$P_b^{D_i}(n+1) = \bar{P}^{D_i}(n+1) - \dot{S}(n+1)/2 . \tag{16}$$

In order to study the dependence of $\dot{E}(n+1)$ and $\dot{S}(n+1)$ on the standard deviation $\gamma(n+1)$, we show the relations between $\gamma(n+1)$ and $\dot{E}(n+1)$ in Fig. 3(a), and between $\gamma(n+1)$ and $\dot{S}(n+1)$ in Fig. 3(b). For estimating the maximum value of $E^{D_d}(n+1)$ of Eq. (13), we vary $x_a^{D_d}$ by 0.0001 because real prices are rounded off after the fifth decimal place. From Fig. 3, we can confirm that when $\gamma(n+1)$ increases, $\dot{E}(n+1)$ and $\dot{S}(n+1)$ increase. In Fig. 3(b), $\dot{S}(n+1)$ increases discretely by a round-off effect. This kind of nonlinearity affects for dealing and dealing time intervals as we will show in Section 3.3.

2.2. The process of dealing execution

In the previous section, we propose a model which decides bid and ask prices by maximizing $E^{D_d}(n+1)$ using the information on the past movement of middle prices. In this section, we model the process of dealing execution in order to decide dealing time intervals. As introduced in the previous section, the distribution of possible future prices of Eq. (2) corresponds to the distribution of the predicted future prices $\bar{P}^{\boldsymbol{D}}(n+1)$ by dealers $\boldsymbol{D}$, $\boldsymbol{D} = \{D_i,\ i=1,2,\ldots,I\}$. Namely, the distribution A of Eq. (2) is rewritten by

$$\bar{P}^{\boldsymbol{D}}(n+1) = \bar{P}(n+1) + \gamma(n+1)\,\mathrm{d}W . \tag{17}$$

As the rule of interbank exchange markets, the quotation by a dealer, who quotes it to other dealers and can really get a deal, is only recorded as a next price. The quotation

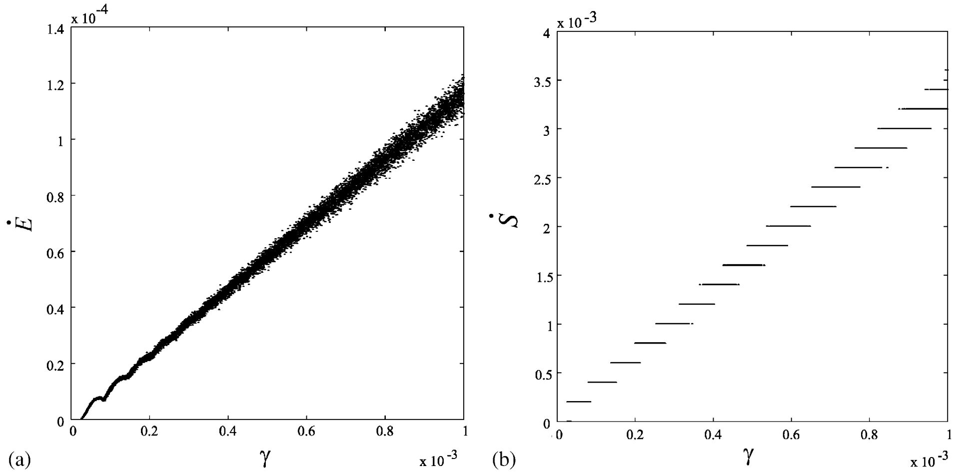

Fig. 3. (a) The horizontal axis is the standard deviation γ of the expected distribution A. The vertical axis is the maximum of expected returns $\dot{E}$. (b) The horizontal axis is the standard deviation of the expected distribution γ. The vertical axis is the decided spread $\dot{S}$.

in fail dealing (rejected by a dealing partner) is not recorded. Namely, we can decide the dealer by referring to a real record of $P(n+1)$. If we denote this dealer by D_l, its predicted future price satisfies $\bar{P}^{D_l}(n+1) = P(n+1)$.

From Eq. (7), an expected distribution (the distribution B' in Fig. 4) of the dealer D_l, who has $P(n+1)$ as a predicted future price, is followed by

$$P^{D_l}(n+1) = P(n+1) + \gamma(n+1)\,\mathrm{d}W \, . \tag{18}$$

From Eqs. (15) and (16), ask and bid prices are decided by

$$P_a^{D_l}(n+1) = P(n+1) + \dot{S}(n+1)/2 \, , \tag{19}$$

$$P_b^{D_l}(n+1) = P(n+1) - \dot{S}(n+1)/2 \, . \tag{20}$$

Moreover, each dealing partner of the dealer D_l has a predicted future price in distribution A of Eq. (17). Then, in order to decide a dealing partner of the dealer D_l, we randomly select a variable from the distribution A. If this selected value, which is the predicted future price of a dealing partner, is larger than the ask price $P_a^{D_l}$ or is smaller than the bid price $P_b^{D_l}$, a dealing really occurs with this dealing partner. Here, the probability of getting a deal $F(n+1)$ at the term $n+1$ is calculated by

$$F(n+1) = f_A(\geqslant P_a^{D_l}(n+1)) + f_A(\leqslant P_b^{D_l}(n+1)) \, , \tag{21}$$

where f_A is the probability distribution function of the distribution A. We iterate this random selection until a dealing really occurs. Once it occurs, we record the number

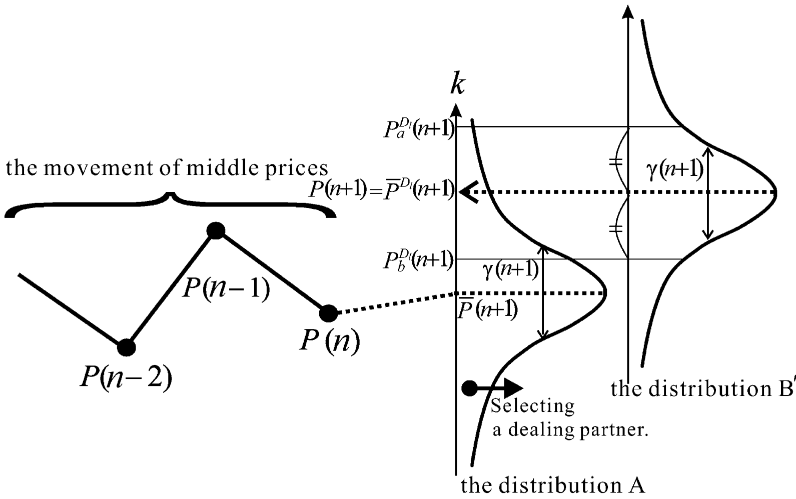

Fig. 4. We remind that the distribution A is the possibility of future prices and corresponds to the distribution of expected future prices of each dealer as shown in Eq. (17) since a future price is decided by the expected price of a dealer who can get a deal. Distribution B' is the expected distribution of future prices by the dealer D_l who quotes an ask price $P_a^{D_l}(n+1)$ and a bid price $P_b^{D_l}(n+1)$ to the other dealers and can really get a deal at the $(n+1)$th term as shown in Eq. (18).

of iterations as the decided dealing time interval $\dot{\tau}(n+1)$. By the above processes, we are able to decide the spread $\dot{S}(n+1)$ and the dealing time interval $\dot{\tau}(n+1)$.

2.3. Approximation of the middle price by ARCH process

Since our aim is to decide dealing time intervals, in order to emphasize the plausibility of this deciding process, we take a best way to use the real data of middle prices for a next price in the previous section. However, it is also important to consider the cases that the real data of the middle prices are missing. Thus, in the present section, we also discuss the plausibility of our model in the case of using a stochastic process instead of the real data.

In conventional studies to model the price movement, stochastic processes have often been applied, such as Auto Regressive Conditional Heteroscedasticity (ARCH) [18] and Generalized Auto Regressive Conditional Heteroscedasticity [19] processes. In this section, we apply ARCH process for discussing the case that we cannot use real movements of middle prices for the proposed model. Thus, even if we do not have real data, we can simulate the mechanism of producing complex behavior of real market data with the proposed model since we need no real data in deciding the dealer D_l of the proposed model.[1]

[1] In the previous section, we discuss how the future spread $\dot{S}(n+1)$ and $\dot{\tau}(n+1)$ can be decided by using the real data of $P(n+1)$ as the future price.

Now, we approximate return rates of middle prices by ARCH process instead of using the real data of $P(n+1)$ in our model. ARCH(1) is described as

$$\sigma^2(n+1) = \beta_0 + \beta_1 x^2(n) , \tag{22}$$

$$x(n+1) \sim N(0, \sigma^2(n+1)) , \tag{23}$$

where $x(n)$ corresponds to the return rate $\Delta P(n)/P(n-1)$. If the variance and the kurtosis of return rates are denoted by σ^2 and κ, respectively, β_0 and β_1 are solved by the following equations [13]:

$$\sigma^2 = \frac{\beta_0}{1-\beta_1} , \tag{24}$$

$$\kappa = 3 + \frac{6\beta_1^2}{1-3\beta_1^2} . \tag{25}$$

These values are evaluated from the real data [20] as $\beta_0 = 0.8147 \times 10^{-7}$ and $\beta_1 = 0.7010$, and are used throughout the present paper. Then, the movement of middle prices is simulated by

$$P(n+1) = P(n)\sqrt{\sigma^2(n+1)}\varepsilon + P(n) , \tag{26}$$

where $\varepsilon \sim N(0,1)$.

3. Comparing the model and the real data

3.1. The property of real data

As the real data, we adopt the time series of tick data between the Swiss franc and the US dollar observed in the interbank market [20]. Figs. 5(a)–(d) show the middle prices, the spread and the dealing time intervals, which are denoted by $\hat{P}$ (dollars), $\hat{S}$ (dollars) and $\hat{\tau}$ (s), respectively. [2] The data series are recorded for 1322 days. In Fig. 5, n indicates discrete time and it increases one by one with every occurrence of dealings.

In order to examine the temporal dependency of the real spread $\hat{S}(n)$, we estimate a power spectrum shown in Fig. 5(e). From Fig. 5(e), we confirm that the real spread $\hat{S}(n)$ has a $1/f^{-2}$ type fluctuation, which means that this temporal dependency is strong. When a probability density function has a power tail, it is well known that the corresponding cumulative distribution function becomes a straight line in a log–log plot. Fig. 5(f) clearly shows that $\hat{S}(n)$ has such a characteristic power law.

Fig. 5(g) shows the relation among the three variables shown below in a three-dimensional state space. They denote absolute values of the difference of middle prices $|\Delta\hat{P}(n)|$, the time interval $\hat{\tau}(n)$, and the spread $\hat{S}(n)$. In Fig. 5(g), there exists the relation that $\hat{\tau}(n)$ becomes small when $\hat{S}(n)$ becomes larger (indicated by the solid

[2] We use "^" for describing real data. For illustration of the proposed model, we use the same variables that do not have any additional symbols.

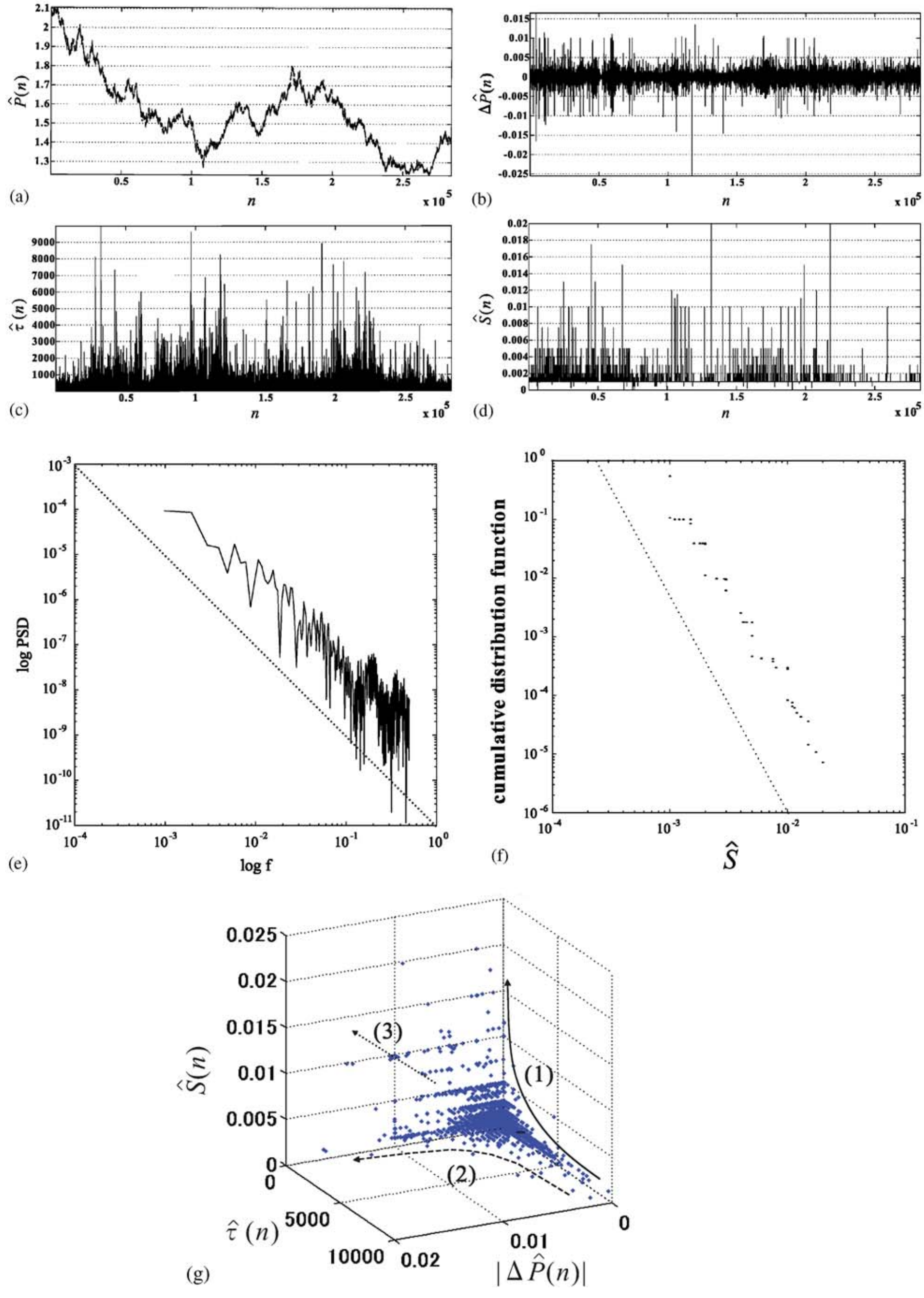

Fig. 5. Real time series of (a) the middle price $\hat{P}(n)$, (b) $\Delta\hat{P}(n) = \hat{P}(n) - \hat{P}(n-1)$, (c) the dealing time interval $\hat{\tau}(n)$ and (d) the spread $\hat{S}(n)$. (e) The power spectrum of $\hat{S}(n)$ in log–log plots. The slope of -2 is plotted by a dotted line. (f) The cumulative distribution function of $\hat{S}(n)$ in log–log plots. The dotted line shows the slope of -4. (g) Relations of $|\Delta\hat{P}(n)|$, $\hat{S}(n)$ and $\hat{\tau}(n)$ in a three-dimensional state space. The correlation coefficient of the dotted arrow (3) is 0.33.

the arrow (1)). Similarly, the dashed arrow (2) shows that $\hat{\tau}(n)$ becomes smaller as $|\Delta\hat{P}(n)|$ becomes larger. Moreover, there exists a slight correlation between $|\Delta\hat{P}(n)|$ and $\hat{S}(n)$ with the correlation coefficient 0.33 (the relation is indicated by the arrow (3) in Fig. 5(g)). In the case of shuffling these variables randomly, the coefficient becomes almost 0, then the value of 0.33 is really significant.

3.2. The property of the data produced from the proposed model

3.2.1. In the case of using the real middle price

First, we utilize the real middle price $\hat{P}$ for the proposed model and calculate the dealing time interval $\hat{\tau}'$ and the spread $\hat{S}'$.[3] We calculate them for 200,000 points with $p = 10$. However, since the movement of $\hat{P}(w)$ $(w = n - p + 1, \ldots, n)$ beyond daily borders is not suitable for our model, we omit such data from our simulation.

We show the time series data obtained from computer simulations in Figs. 6(a) and (b). Figs. 6(c) and (d) show that $\hat{S}'$ has the same statistical properties as the real data $\hat{S}$ has. Moreover, Fig. 6(e) also shows that there is a similarity between the real data and the produced data from the proposed model in a state space reconstruction. There exists some correlation between $|\Delta\hat{P}(n)|$ and $\hat{S}'(n)$ whose correlation coefficient is 0.32 (the relation is indicated by the arrow (3) in Fig. 6(e)).

3.2.2. In the case of using the middle price simulated by ARCH process

Next, we make a simulated middle price $\tilde{P}(n)$ by ARCH(1) in Fig. 7(a) with $\beta_0 = 0.8147 \times 10^{-7}$ and $\beta_1 = 0.7010$. We show the change $\Delta\tilde{P}(n)$ in Fig. 7(b). Then, we utilize $\tilde{P}(n)$ for the proposed model by $p = 10$, and calculate the time series data of $\tilde{\tau}'(n)$ and $\tilde{S}'(n)$ that have total 200,000 points. We show these time series data in Figs. 7(b) and (c) and show statistical properties of $\tilde{S}'$ in Figs. 7(e) and (f). In Figs. 7(e) and (f), we observe almost the same power law as shown in Figs. 5(e) and (f) and Figs. 6(c) and (d). Fig. 7(g) shows the relation among the three variables in the three-dimensional state space. In the same way as in Figs. 5(g) and 6(e), we have very similar results between the real data and our model, namely there is a sort of dependency indicated by the arrows (1)–(3). Moreover, there exists some correlation between $|\Delta\tilde{P}(n)|$ and $\tilde{S}'(n)$ with the correlation coefficient 0.28 (the relation is indicated by the arrow (3) in Fig. 7(g)).

3.3. Discussion on the relation in the state space

In Fig. 5(g), we discover that there exists a sort of dynamical relation among $|\Delta\hat{P}|$, $\hat{S}$ and $\hat{\tau}$. In this section, we discuss why such relations are observed in the results.

First, arrow (1), the relation between $\hat{S}(n)$ and $\hat{\tau}(n)$, is caused by the effect that real prices, including spreads, are rounded off after the fifth decimal place. The reason is

[3] We use an extra symbol "′" to each variable, which means that these variables describe the data produced from the proposed model.

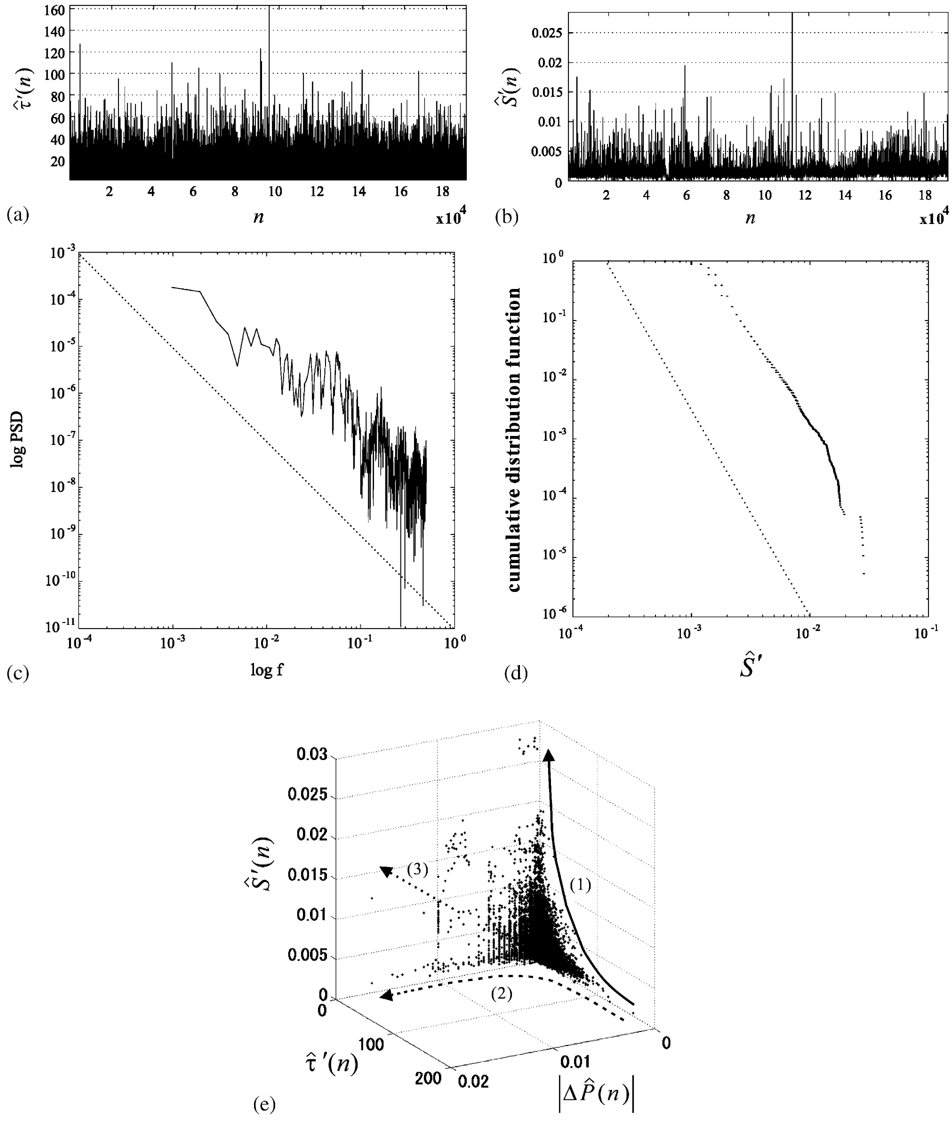

Fig. 6. (a) The time series data of $\hat{\tau}'(n)$ and (b) $\hat{S}'(n)$ produced by the proposed model with the information of real data. (c) The power spectrum of $\hat{S}'(n)$ in log–log plots. The slope of -2 is plotted by a dotted line. (d) The cumulative distribution function of $\hat{S}'(n)$ in log–log plots. The dotted line shows the slope of -4. (e) In the case of using $|\Delta\hat{P}'(n)|$, $\hat{S}'(n)$ and $\hat{\tau}'(n)$. The correlation coefficient of the dotted arrow (3) is 0.32.

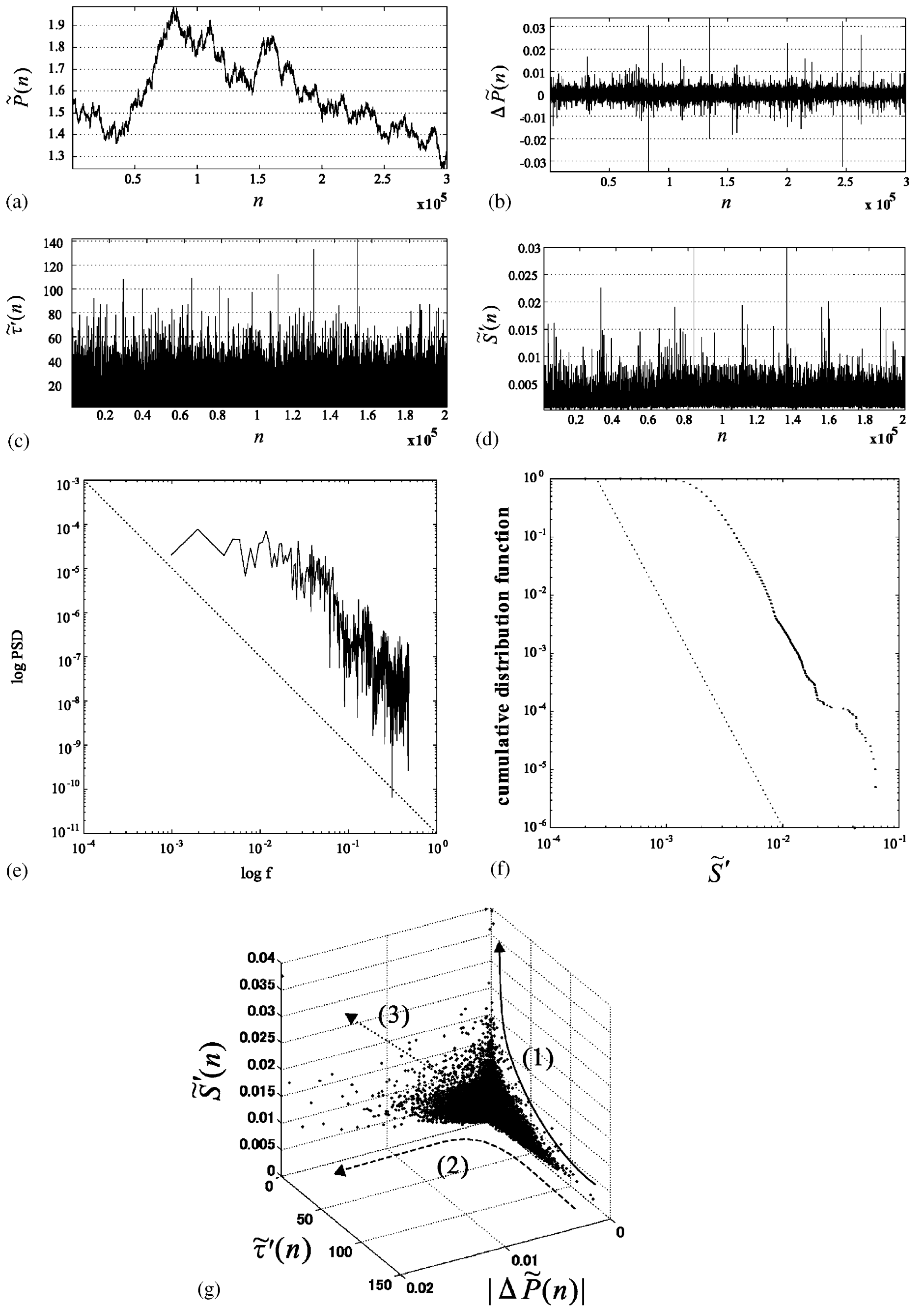

Fig. 7. (a) The time series data of $\tilde{P}(n)$ produced by ARCH(1). $\beta_0 = 0.8147 \times 10^{-7}$ and $\beta_1 = 0.7010$. (b) The time series data of $\Delta\tilde{P}(n)$. (c) The time series data of $\tilde{\tau}'(n)$ and (d) $\tilde{S}'(n)$ produced by the proposed model. (e) The power spectrum of $\tilde{S}'(n)$ in log–log plots. The slope of -2 is plotted by a dotted line. (f) The cumulative distribution function of $\tilde{S}'(n)$ in log–log plots. The dotted line shows the slope of -4. (g) In the case of using $|\Delta\tilde{P}'(n)|$, $\tilde{S}'(n)$ and $\tilde{\tau}'(n)$. The correlation coefficient between $\hat{\tau}'(n)$ and $\hat{S}'(n)$, the dotted arrow (3), is 0.28.

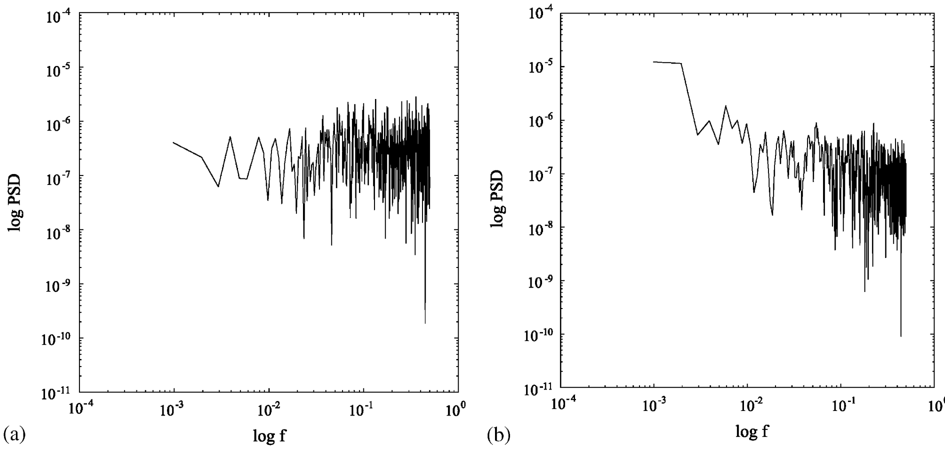

Fig. 8. (a) The power spectrum of $\Delta\hat{P}(n)$ in a log–log plot and (b) the power spectrum of $|\Delta\hat{P}(n)|$ in a log–log plot.

as follows. In the present paper, we simulate $\dot{S}(n+1)$ by 0.0001 like a real dealing (as is explained in Section 2.1). Let us denote the rounding error of $\dot{S}(n+1)$ by $\pm\xi(n+1)$ and denote the continuous best spread by $\acute{S}(n+1)$. Then, Eqs. (19) and (20) are rewritten as

$$P_a^{D_l}(n+1) = P(n+1) + (\acute{S}(n+1) \pm \xi(n+1))/2 , \tag{27}$$

$$P_b^{D_l}(n+1) = P(n+1) - (\acute{S}(n+1) \pm \xi(n+1))/2 . \tag{28}$$

In calculating the occurrence probability of a dealing, $F(n+1)$, in Eq. (21), the rounding error strongly affects on $F(n+1)$ when the standard deviation $\gamma(n+1)$ is small, because the distribution A is a normal one. Moreover $F(n+1)$ increases by $-\xi(n+1)$ or decreases by $+\xi(n+1)$. When $\acute{S}(n+1)$ is small, that is, the standard deviation $\gamma(n+1)$ is small, the decrease of $F(n+1)$ by $+\xi$ is much affected. Moreover, because the dealing time interval $\dot{\tau}(n+1)$ is affected by $F(n+1)$, the term $+\xi$ leads to the reduction of $\dot{\tau}(n+1)$. To summarize, for the realization of large $\dot{\tau}(n+1)$, it is necessary that the quantity $\dot{S}(n+1)$ $(=\acute{S}(n+1) \pm \xi(n+1))$ is small. That is, $\acute{S}(n+1)$ is small as well.

Next, we explain the relation between $|\Delta\hat{P}(n)|$ and $\hat{\tau}(n)$ indicated by arrow (2). Because the power spectrum of $\Delta\hat{P}(n)$ shows almost the same property as the white noise (Fig. 8(a)), $\mu(n) \simeq 0$ and $\bar{P}(n) \simeq P(n-1)$ in Eq. (3). By substituting these relations into Eq. (17),

$$\bar{P}^{\boldsymbol{D}}(n) \simeq P(n-1) + \gamma(n)\,\mathrm{d}W . \tag{29}$$

Moreover, since the region R_A (in which the probability of dealing realizations of distribution A increases) is larger than the region R_B (in which the probability of dealing realizations of distribution A decreases) as the movement of the middle price

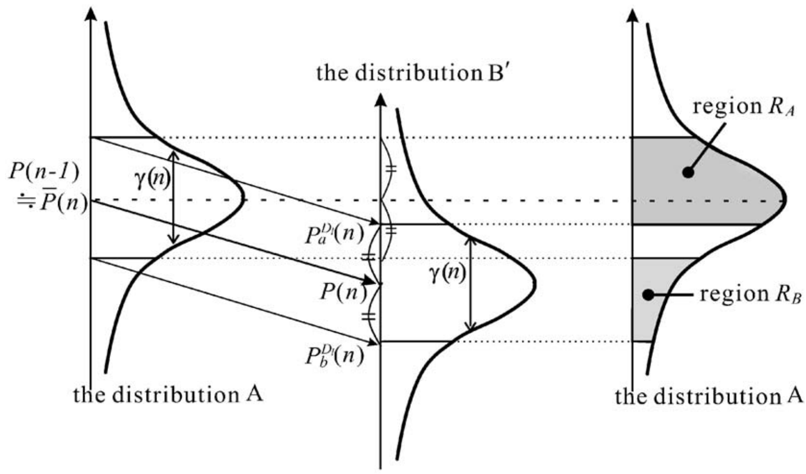

Fig. 9. R_A is a region in which the occurrence probability of a dealing increases. R_B is a region in which the occurrence probability of a dealing decreases.

$|\Delta P(n)|$ fluctuates, it is likely to realize a dealing in a shorter term with small $\dot{\tau}(n)$ (Fig. 9). Namely, it is sufficient to reduce $\dot{\tau}(n)$ that $|\Delta P(n)|$ becomes large.

Finally, we discuss the relation between $|\Delta\hat{P}(n)|$ and $\hat{S}(n)$ indicated by arrow (3). First, recall that the spread $\dot{S}(n)$ depends on the middle prices in the last p terms $\{P(n-p),\ldots,P(n-1)\}$. When the movement of middle prices in the last p terms is large, $\dot{S}(n)$ becomes large as well. Since $|\Delta\hat{P}(n)|$ has slightly a temporal dependence as shown in Fig. 8(b), $|\Delta P(n)|$ could take large values too. Namely, there exists a positive correlation between $|\Delta P(n)|$ and $\dot{S}(n)$. In this reason, we can understand why there exists the relation among $|\hat{P}(n)|$, $\hat{\tau}(n)$ and $\hat{S}(n)$ as shown in Fig. 5(g).

4. Verification of the anomaly with the interbank exchange model

In our previous study [15], we discovered a sort of anomaly from the real data as discussed in Section 1. In this section, we show first that such an anomaly could also appear in the data produced by the proposed model. Next, we discuss the reason why such an anomaly occurs from the viewpoint of the proposed model.

4.1. Raster plot and PSTH of the data obtained by the proposed interbank exchange model

At the beginning, we briefly review our previous results [15]. In Ref. [15], in order to analyze interactions of the three variables ($P(n), S(n)$ and $\tau(n)$), we use raster plots and peri-stimulus time histograms (PSTH) [21–23]. These methods are utilized for evaluating the statistical property of neural spike timings in the field of neurophysiology and provide good schemes to represent spike timings visually.

For example, in order to analyze a response of neural spikes caused by external stimuli, observed spikes by several trials are plotted along a horizontal axis. In this case, these spikes are aligned on each line with adjusting the timing of external stimuli. It is called a raster plot. If there exists a temporal tendency in spike timings, we can visually recognize it by the raster plots. For further understanding of the ensemble behavior of spike timings, PSTH can be calculated by transforming raster plots into a histogram expression.

Although the above methods are originally proposed in the field of neurophysiology, we can expect that both methods can be applied to analyze interbank exchange market data since there is a similar aspect between neural spikes and interbank exchange market data from the viewpoint that the timings of each occurrence might have essential information. However, there is a significant difference between them. For neural spikes, it is widely acknowledged that there is no information in intensities of each spike, because the intensities of spikes become the same due to all-or-nothing property when they propagate through axons. On the other hand, for interbank exchange market data, the intensities of each occurrence (spike) have essential information about the prices. Thus, we must analyze the ensemble behavior of the intensities of price movements as well as timing. Considering the similarity and the difference between these two classes of spike information, we modify the methods to be suitable for analyzing interbank exchange market data.

In drawing raster plots, we consider the occurrence of actual dealings as spikes, and each day as a single trial. We also consider the time at which the spread becomes very large as time at which external stimuli are applied in order to investigate the response of market from the movement of spread reflecting dealers' mind. In addition, since the response might depend on a longer temporal effect due to daily activity, we define four temporal sections by dividing the daily dealing time from 9:00 a.m. to 5:00 p.m. (each section has 2 h long). Thus, we calculate raster plots and PSTHs by the following data selection schemes, (i) and (ii), and investigate the existence of response from external stimuli by comparing both of their results:

(i) In the case that external stimuli are not applied.
    We randomly select 15 dealing data, in which the actual dealings occur nearly at the median of each section without any prior information. Here, at the times when these dealings occur, the movements of spreads are regular. Then, their dealings are placed on the vertical line at $t=0$ on the horizontal axis in raster plots (though the selected 15 dealings are not considered to be applied external stimuli). Here, t describes physical continuous time.

(ii) In the case that external stimuli are applied.
    We select top 15 dealing data series whose spreads become much larger in each section. Namely, these dealings are treated as if external stimuli were applied to them. Then, these dealings are placed as the same as in (i).

If the movement of spreads has no relation with the dealers' action and does not stimulate the dealers' action, there is not so large a difference between the results by these two cases that external stimuli are (i) not applied and (ii) applied. However, by the results of method (ii), we observe that the dealing time interval becomes shorter. In addition, the price movement also shows the difference that it has a peak in the temporal bin at $t=0$, which means that the expansion of spreads makes the movement of middle prices large.

It should be noted that if there exists any expansion of spreads, it indicates that the ask and bid prices differ from the middle price. Thus, dealers try to sell at higher prices and/or to buy at lower prices. It is very natural to consider that since it is hard for such greedy dealers to find a dealing partner, the dealing time interval becomes longer. In spite of such bull quotations, the dealing time interval becomes shorter. We call it an anomaly. We also observe such tendency for the other temporal sections. Namely, it is shown that the expansion of spreads often makes the interval time of dealings shorter and the movement of prices larger.

Now, in order to show that such an anomaly could also appear from the data obtained by the proposed model, we conduct the following two experiments I and II:

(I) We apply the middle prices of real data to the proposed model in order to obtain dealing time intervals and spreads.
(II) We apply the middle prices simulated by an ARCH process to the proposed model in order to obtain dealing time intervals and spreads.

We set the temporal bin width as about 30 steps in order to fit the average values of PSTH in the previous study [15]. In the jth temporal bin ($j=1,\dots,J$), dealings are denoted by $s_j(m)$ ($m=1,\dots,M_j$) as shown in Fig. 10, and the average amount of the daily dealing in each temporal bin is calculated by

$$H_j = \frac{M_j}{D}\,, \tag{30}$$

where D is the number of selected dealing data series in methods (i) and (ii). Thus, we set $D=15$. Moreover, in order to examine the average behavior of the movement of middle prices, the movement of the middle price $\Delta P_{s_j(m)}$ in the dealing $s_j(m)$ is used for calculating the following histogram:

$$h_j = \frac{1}{M_j}\sum_{m=1}^{M_j} |\Delta P_{s_j(m)}|\,, \tag{31}$$

where $\Delta P_{s_j(m)} = P_{s_j(m)} - P_{r_j(m)}$, $P_{r_j(m)}$ is a one-step previous price of $P_{s_j(m)}$ and $r_j(m)$ is a one previous dealing of $s_j(m)$ as shown in Fig. 10. In Fig. 10, since one previous dealing of $s_j(m+1)$ exists in the same bin, $r_j(m+1)=s_j(m)$. However, if a previous dealing of $s_j(m)$ does not exist in the same bin, we set its previous value $r_j(m)$ in the previous bin (the $(j-1)$th bin). Then, $r_j(m)\neq s_j(m-1)$.

4.2. Appearance of the anomaly

Fig. 11 shows the analysis results. In Figs. 11(a) and (b), we show the results in the case of Experiment (I) (using real data). In Figs. 11(c) and (d), we show the results in the case of Experiment (II) (using ARCH). With each case of applying the middle prices of real data (in Figs. 11(a) and (b)) and simulated data by ARCH

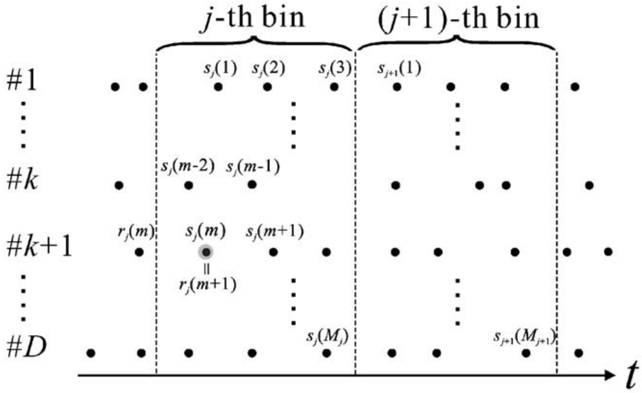

Fig. 10. A raster plot for calculating Eqs. (30) and (31). In the jth bin, the number of dealings is M_j. Then, all dealing timings are numbered from 1 to M_j with the method of raster scanning frequently used in image processing. In the figure, $r_j(m)$ is one step previous dealing of $s_j(m)$. Here, each sequence of dealings is aligned horizontally. Since one step previous dealing of $s_j(m+1)$ exists in the same bin, $r_j(m+1) = s_j(m)$. However, in the case that one step previous dealing of $s_j(m)$ does not exist in the same bin, we set its previous value $r_j(m)$ from the previous bin (the $(j-1)$th bin). Then, $r_j(m) \neq s_j(m-1)$.

process (in Figs. 11(c) and (d)), we can confirm appearance of the anomaly that the expansion of spreads makes the interval times short and the movements of middle prices large. In order to discuss the reason for the appearance of the anomaly, it is useful to use the discussion on the relation among $\hat{P}$, $\hat{S}$ and $\hat{\tau}$ indicated by arrows (1)–(3) in Section 3.3.

First, to discuss the anomaly in h_j, we reconsider the relation indicated by arrow (3) in Figs. 5(g), 6(e) and 7(g). We should remember that the spread $\dot{S}(n)$ depends on the movement of the middle prices for the last p terms, $\{\hat{P}(n-p), \ldots, \hat{P}(n-1)\}$. Therefore, by weak temporal dependence of $|\Delta\hat{P}|$ shown in Fig. 8(b), there exists positive correlation between $S(n)$ and $|\Delta P(n)|$. However, when $S(n)$ becomes larger, this anomaly appears. In the proposed model, only the last movement of $|\Delta\hat{P}(n-1)|$ makes $\dot{S}(n)$ larger. Thus, since the memory of the last price movement remains in the temporal dependency of $|\Delta\hat{P}|$, when $\dot{S}(n)$ becomes larger, the peak showing the increase of h_j appears clearly.

Next, we discuss the anomaly in H_j. The anomaly is easily explained by the same discussion as we used for the relation between $|\Delta P(n)|$ and $\tau(n)$ indicated by arrow (2). Since the dealing time interval $\dot{\tau}(n)$ becomes short when the price movement becomes large, it is very clear that the dealing time interval becomes short from the relation shown by arrow (2).

4.3. Disappearance of an M type rhythm

In the previous study [15], when we use the first scheme (i), the amount of the daily dealing shows a temporal rhythm whose peak is at about 9:00 a.m. and 3:00 p.m., which resembles a letter “M” (price movements are almost flat (no particular rhythms)). On the other hand, from the results obtained by method (ii), we have found that many

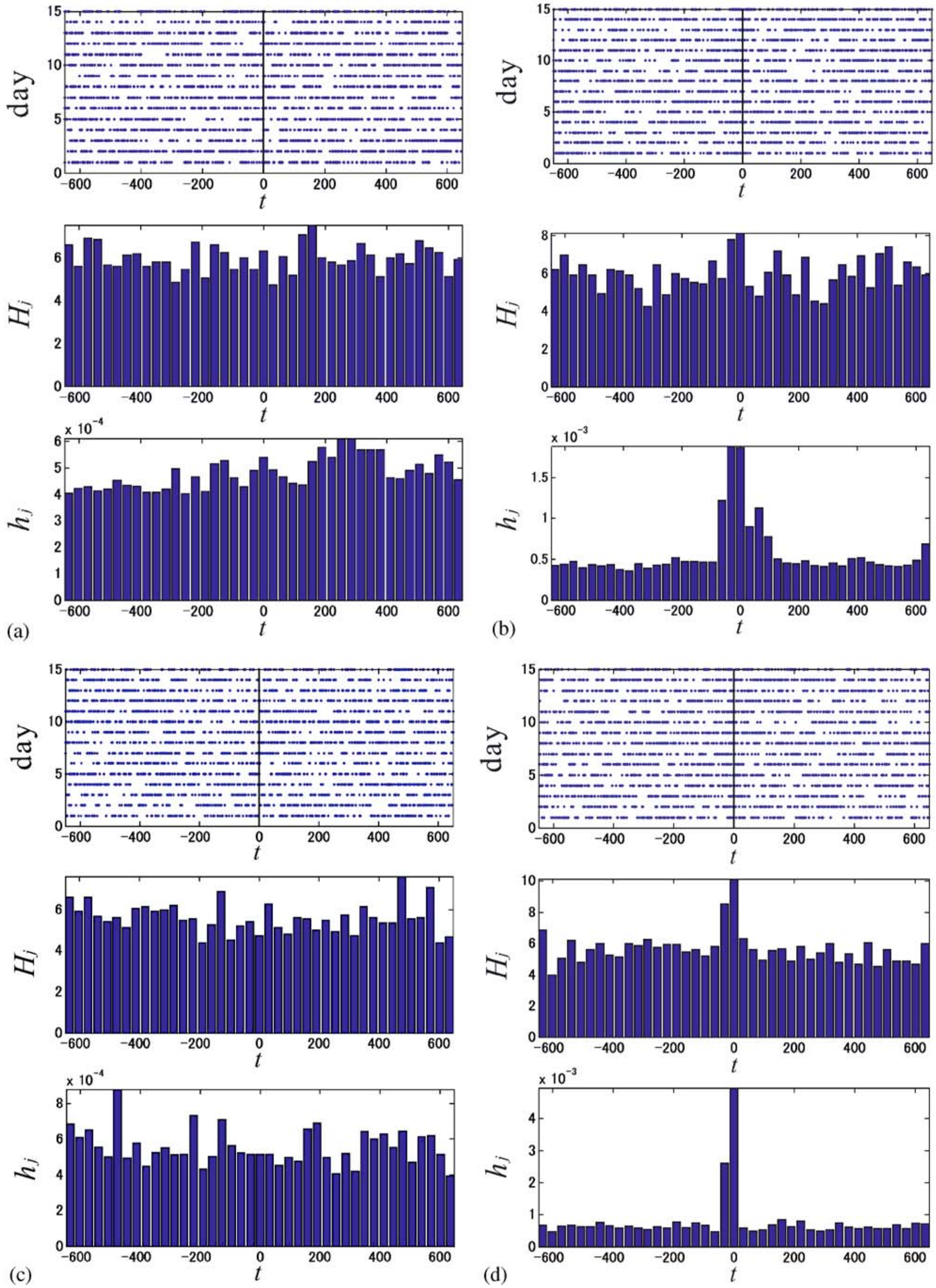

Fig. 11. The results (a) by method (i) and Experiment (I) (using real data), (b) by method (ii) and Experiment (I), (c) by method (i) and Experiment (II) (using ARCH process) and (d) by method (ii) and Experiment (II).

dealings appear in temporal bins near $t = 0$ and the large amount of dealings destroys the rhythm of the shape "M." Although we can explain why the anomaly occurs in real data by our model, an M type rhythm cannot be observed in Fig. 11(a).

We discuss the reason as follows. In real situations, the number of dealings depends on the number of dealers who participate in the market. This number always fluctuates in the real market. For example, since there exists a difference between opening time and closing time in each bank, the number of dealers who participate in the market gradually increases in the early morning and it gradually decreases in the evening. Moreover, the number becomes very small at noon because of lunchtime. Namely, since the basic bias, the average number of real dealings in this case, fluctuates, such an M-type rhythm appears. However, in the proposed interbank exchange model, since we do not consider such a large temporal trend for simplification, the M-type rhythm does not appear. However, as in our previous study [15], when the spread becomes large ($t = 0$), we can confirm the reduction of dealing time intervals and the expansion of the movement of prices. Figs. 11(c) and (d) show the results of Experiment (II). The same phenomenon can also appear in this case.

5. Conclusions

In the present paper, we have proposed a novel model of interbank exchange markets on the basis of the three important variables, namely, dealing time intervals, spreads and price movements. To confirm the proposed model, we have conducted numerical simulations on the real data and the ARCH process (as the movement of middle prices). We have shown that the outputs from the proposed model have almost the same statistical properties as the real data have. From the viewpoint of the reconstructed state space, raster plots and PSTHs, we have also shown that there exists a possible dynamical relation among the simulated three variables, namely, the movement of prices, the dealing time interval and the spread. The results shown in the present paper strongly suggest the plausibility of the proposed model. Namely, this model can make us understand the characteristic phenomena which are observed in actual interbank exchange markets and it reproduces complex behavior of price movements. The present results are also supported by the fact that the statistical properties of three variables are preserved.

Moreover, the other motivation of the present paper is to discuss the anomaly discovered in our previous study [15]. Using the same discussion for the existence of the relation among these three variables, we have explained the anomaly on the basis of our proposed model. In addition, we have also shown that we can reproduce the anomaly that is often observed in the real interbank exchange markets, using our proposed model by producing the simulated time series of the interbank exchange markets.

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