Multivariable Modeling on Complex Behavior of a Foreign Exchange Market

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Summary. We discover a remarkable property of a foreign exchange market: when the spreads, the difference between ask and bid prices, become large, the dealing time intervals become short and the price movements become strong. To discuss an interaction of these variables, we propose a model on complex behavior of the foreign exchange market. Then, we confirm the plausibility of the proposed model by comparing the statistical properties of the real market and the model. Finally, we discuss why there exists the interaction on the basis of our model, and clarify the mechanism for producing complex behavior with the interaction.

Key words: foreign exchange market, spreads, dealing time intervals, middle price, raster plots, peri-stimulus time histograms

1 Introduction

The present paper mainly focuses on the relation between the dealing time intervals and the spreads, the difference of bid and ask prices, which could affect the price movements for the following reason. Since dealers generally wait for an opportunity of getting large earnings, the dealing time interval which corresponds to dealing timings and the spread may provide important information to dealers' mind getting earnings and hedging risks and directly link to dealers' motivation for getting a deal. Thus, these variables could be essential factors for modeling exchange markets.

To confirm the interaction among these variables, we introduce novel analysis tools which are based on raster plots[1] and peri-stimulus time histograms (PSTH)[2]. These methods are popular in the field of neurophysiology and are suitable to observe ensemble behavior. Namely, these methods are useful for visually understanding the complex behavior of multivariables in foreign exchange markets. From analysis results, we see that when the spreads become large suddenly, the dealing time intervals become short and the price movements become large. Since the expansion of the spread means that ask and bid prices are separated from the middle price, it is natural to consider that the dealer tries to sell at higher prices and to buy at lower prices. Such a bull quotation is anticipated to lead the situation that dealing time intervals become longer since it is not so easy to find a dealing partner. However, our analyses show completely opposite tendencies.

In order to understand the mechanism of producing such unexpected behavior in a foreign exchange market, we propose a plausible model of explaining the behavior in the foreign exchange market. Then, from the viewpoint of the proposed model, we clarify the reasons why there exists the singular interaction among these three variables in the real market.

2 Analyzing Real Dealing Data

For analyses, we use the time series of tick data between the U.S. dollar and the Swiss franc observed in an interbank market[3]. The tick data set is recorded from January 1986 to April 1991 (total 1,322 days), and the total number of the data points is 282,956. Generally, a tick data set has the following intrinsic aspects. There is a sort of discontinuity on dairy tick data, because banks close at nights, weekends and holidays. Then the first and the last dealing times are different from each other day by day. The dealing-time stamps are recorded at actual timings when every deal occurs.

Now, we denote the temporal index by n, which indicates a discrete time and increases one by one whenever each dealing occurs. Moreover, we denote each middle price, namely a mean value of bid and ask prices, by $P(n)$ [dollar] and their temporal differences by $dP(n)$ $(= P(n) - P(n-1))$, respectively. We also denote the spread, the difference between bid and ask prices, by $S(n)$ [dollar] and the dealing time interval by $\tau(n)$ [sec].

First, we analyze the interaction of three variables by drawing raster plots (RPs)[1] and peri-stimulus time histograms (PSTHs)[2]. These methods are often utilized for analyzing neural spikes in the field of neurophysiology, and are good schemes to express spike timings visually. In the case of analyzing neural spikes, every observed datum is plotted in horizontal lines with external stimuli. However, for analyzing financial indices, it is not easy to consider the existence of external stimuli explicitly. Then, we propose a modified method in order to apply these techniques to analyze interbank data.

For drawing RPs, we consider each day as each trial, and occurrences of actual dealings as spikes. Since it is very rare to observe the case in which spreads become very large, we assume that a particular time at which spreads become very large is equal to the time at which external stimuli are applied in order to investigate alteration in the market's behavior by the external stimuli.

For evaluating PSTHs, we have to consider how to treat the variation of opening and closing times in each day. In the i-th temporal bin $(i = 1, 2, \cdots, B)$, deals are denoted by $s_i(k)(k = 1, \cdots, N_i)$. When there are d_i days in which the start of dealing is later and the finish of dealing is faster than the median of the corresponding temporal bins, the average of the dairy dealing in the i-th temporal bin is calculated by $H_i = N_i/(D - d_i)$, where D is the number of total days. Moreover, in order to examine an ensemble behavior of the movement of middle prices, the difference of the middle price in the dealing $s_i(k)$ is used for calculating the following histogram, $h_i = \frac{1}{N_i}\sum_{k=1}^{N_i} |\Delta P_{s_i(k)}|$, where $\Delta P_{s_i(k)} = P_{s_i(k)} - P_{r_i(k)}$, and $P_{r_i(k)}$ is a one-step previous price of $P_{s_i(k)}$. Then, we divide the total dealing length from 9:00 a.m. to 5:00 p.m. into four sections (each has two hours long) and classify all tick data into each section on the basis of the dealing time.

Calculating RPs and PSTHs by the following two methods (i) and (ii), we compare both of these results.

(i) We randomly select 15 dealings, in which the actual dealings occur nearly at the median of each section, and we can temporarily identify their dealing time as the time of external stimuli. Then, their deals are aligned with the vertical line at $t = 0$ on the horizontal axis in RPs. Here, t is continuous time.
(ii) We select top 15 dealing data sets whose spreads become much larger from each temporal section. Stimuli are considered to be the same as those in (i).

If the movement of spreads has no relation to the dealer's action, we expect that no significant difference appears between the results obtained by the above two methods.

Figure 1 shows the results obtained by the above analyses. The temporal section is from 11:00 a.m. to 1:00 p.m, $B = 40$ (the duration of the temporal bin is about 1,000 [sec]) and $D = 15$. In Fig.1(a), the histogram H_i shows a kind of temporal rhythm (a shape of the letter "M") and the histogram h_i is almost flat (no particular rhythm). The M type rhythm shows a daily trend of the dealing time intervals which depend on the number of dealers who participate in the market. Since there are few dealers in early morning, lunch time and evening, the dealing time intervals tend to become larger in such time regions. Thus, H_i becomes shorter and the M type rhythm appears. On the other hand, from Fig.1(b), we find that many dealings appear in temporal bins near $t = 0$ and the large amount of dealings destroys the rhythm of the shape "M," then the dealing time interval becomes shorter. Moreover, the histogram h_i also shows that the movement of middle prices has a peak in the temporal bins at $t = 0$, which means that the expansion of the spreads makes the movement of middle prices larger.

The term *expansion of the spread* means that the bid and ask prices separate from the middle price, which indicates that dealers try to buy at lower prices and to sell at higher prices. It is very singular, because in spite of such bull quotations, this dealing time intervals become shorter as introduced

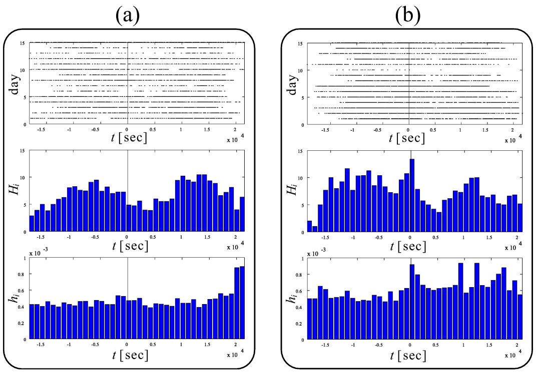

Fig. 1. Top plots are modified RPs, and middle and bottom plots are modified PSTHs. Figures (a) and (b) correspond to the methods (i) and (ii), respectively.

in Sec.1. Such tendencies are also observed if we use other sections (9:00 a.m.∼11:00 a.m., 1:00 p.m.∼3:00 p.m. and 3:00 p.m.∼5:00 p.m.) as the central time.

Moreover, from Refs.[4, 5], we obtain the result that prediction accuracy of middle-price movements is better by using these three variables than using only the middle price. The existence of the interaction among three variables is also supported by this improvement on prediction accuracy.

3 A Model for a Foreign Exchange Market

In this section, we propose two models of the interbank exchange dealing, which incorporate the following three variables, dealing time intervals, spreads and middle-price movements, in order to discuss the singular interaction among these variables.

Model 1: a mechanism for deciding bid and ask prices by each dealer
We calculate a distribution of possible future prices according to past middle-price movements during last p terms, $\{P(n-p+1), \cdots, P(n)\}$. The distribution is based on a geometric Brownian motion[6, 7], and its standard deviation temporarily changes according to the past movements. Moreover, though its mean value also changes, it is almost equal to $P(n)$[8]. Since a future price is decided by a dealer's quotation, possible future prices correspond to predicted future prices by all dealers. Since each dealer does not know the predicted future prices by the others, the dealer predicts them and evaluates the expected gain and the risk by prediction errors. Each dealer estimates the expected return by the expected gain and risk and decides best bid and ask prices, that is a best spread $\dot{S}(n+1)$, by maximizing the expected return.

As one of the characteristics of Model 1, when past middle prices fluctuate heavily, the standard deviation of the distribution of possible future prices becomes large. Namely, the dealers' quotations are distributed widely. In this case, even if a dealer quotes a bull quotation, he can find a dealing partner. Thus, a best spread tends to be large.

Model 2: describing a dealing process for deciding dealing time intervals First, we select a dealer A who can really get a deal by referring to a real future price $P(n+1)$. In this model, we mainly consider how we can model the behavior of spreads and dealing time intervals. Then, we use real future prices for the present purpose. However, even if we do not have real prices, we can substitute the ARCH model[9] or the GARCH model[10] for real prices[8]. In Model 1, we know this dealer's quotation and the distribution of predicted future prices by the other dealers. Next, we randomly select a dealing partner of the dealer A and iterate this random selection until a dealing really occurs, that is the predicted price of a dealing partner is larger than the ask price or is smaller than the bid price of the dealer A. Once it occurs, we record the number of iterations as the decided dealing time interval $\tau(n+1)$. By iterating the above processes, we are able to decide both of the spreads and the dealing time intervals.

In order to confirm the plausibility of our models, we analyze whether or not the data obtained from our models have similar properties to the real data[8]. By this estimation, we confirm that the data obtained by the model have almost the same statistical properties as the real data. In particular, each power spectrum and each cumulative distribution function have the same power laws of same slopes. Moreover, by calculating RPs and PSTHs, the same singular interaction is confirmed from the data obtained by the our model as well[8]. From these results, we can consider that our model has very good plausibility for explaining a producing mechanism of complex behavior observed in real markets.

4 Discussion on the singularity of h and H

Once the spread becomes larger, the middle-price movement is large, that is, the singularity on h appears. In Model 1, since a best spread $\dot{S}(n+1)$ depends on the past middle-price movements for last p terms, $\{P(n-p+1), \cdots, P(n)\}$, only the last price movement $|dP(n)|$ makes $\dot{S}(n+1)$ larger than $\dot{S}(n)$. Moreover, by analyzing real data, we can confirm that $|dP|$ has a slight temporal dependency[8]. Thus, when $S(n+1)$ becomes larger, since the fact that the last price movement $|dP(n)|$ is large affects to the temporal dependency of $|dP(n{+}1)|$, then $|dP(n{+}1)|$ becomes large. By above discussion, we can explain the reason why the singularity on h appears.

In Model 2, when $|dP(n+1)|$ is large, the dealer A, who can really get a deal, puts his own middle price near to the tail of the distribution of the predicted future price by all dealers as shown in Fig.2(a). Figure 2(b) is the situation when the $|dP(n+1)|$ is small. If $|dP(n+1)|$ is large (Fig.2(a)) rather than when the $|dP(n{+}1)|$ is small (Fig.2(b)), there exists more dealing

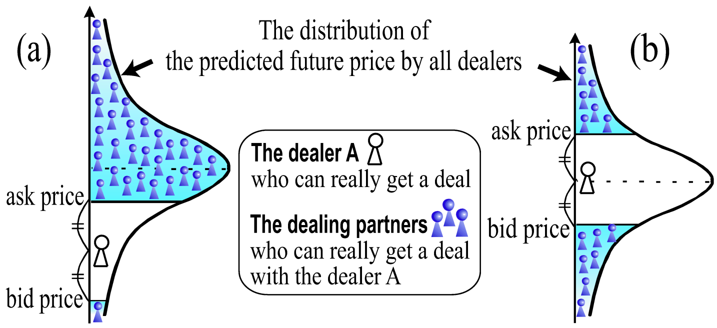

Fig. 2. Relations between a dealer and its dealing partners. The mean value of each distribution is almost equal to $P(n)$.

partners who can really get a deal with the dealer A[8]. Therefore, in the situation as Fig.2(a), it is easy to find a dealing partner randomly in Model 2 for a few numbers of iteration. Thus, as $|dP(n+1)|$ is large, the dealing time interval $\tau(n+1)$ becomes shorter. Thus, we can explain the reason why the singularity on H appears.

5 Conclusions

By using modified RPs and PSTHs, we discover the singular interaction among the three important variables, the middle-price movements, the dealing time intervals and the spreads, in a real foreign exchange market. In order to confirm the mechanism of such interaction among these variables, we introduce a plausible model with multivariables. We also mention why there exists the singular interaction on the basis of our model.

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